ФИЗИКА

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CHIRAL PARAMETRIZATION OF QCD VECTOR FIELD. GLUON SECTOR

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The chiral parametrization of QCD gauge field is considered in details in an approach developed earlier for $SU(2)$ and $SU(3)$ cases. A color chiral field is introduced, gluons are chirally rotated, and vector component of rotated gluons is defined on condition that no new color variables appeared with the chiral field. This condition for $SU(3)$ case associates such a vector component with $SU(3)/U(2)$ coset plus an $U(2)$ field. The QCD vector field in CP^2 and $U(2)$ sectors is studied in new variables of chiral parametrization. The singlet gluonium can acquire mass due to formation of dimension two vacuum condensate of induced axial-vector field. Refs 34.

Keywords: gluons, quantum chromodynamics, Cho—Faddeev—Niemi—Shabanov (CFNS) decomposition.

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КИРАЛЬНАЯ ПАРАМЕТРИЗАЦИЯ ВЕКТОРНОГО ПОЛЯ КХД. СЕКТОР ГЛЮОНОВ

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Киральная параметризация калибровочного поля КХД подробно рассмотрена в подходе, предложенном ранее для $SU(2)$ и $SU(3)$ случаев. Введено цветное киральное поле, глюонные поля кирально повёрнуты, и векторная компонента повёрнутых глюонов определена условием отсутствия излишних цветных степеней свободы, вносимых киральным полем. Для случая $SU(3)$ это условие ассоциирует векторную компоненту с комплексным проективным пространством $SU(3)/U(2)$ плюс $U(2)$ поле. Векторное КХД поле рассмотрено в секторах CP^2 и $U(2)$ в новых переменных киральной параметризации. Синглетный глюоний может получить массу вследствие образования вакуумного конденсата размерности 2 наведённых аксиально-векторных полей. Библиогр. 34 назв.

Ключевые слова: глюоны, квантовая хромодинамика, разложение Чо—Фаддеева—Ниеми—Шабанова.

Dedicated to Yu. V. Novozhilov on his 90th birthday

Introduction. The monopole condensation scenario is considered as the most probable way to confinement [1–4]. Therefore, much efforts were undertaken last decade, in order to find a proper parametrization of the gauge field in QCD, which could contain necessary

topological properties. Long ago, the Faddeev soliton picture [5] of QCD excitations was the first model, where a unit three-dimensional vector $\boldsymbol{n}(x)$, $\boldsymbol{n}^2 = 1$, was introduced to describe knot solitons in four-dimensional space-time. In order to consider magnetic properties of the QCD Cho [6, 7] and independently Duan and Ge [8] used the $\boldsymbol{n}(x)$ field, as a vector in the isospin space, to build the connection $[\boldsymbol{n}, \partial_\mu \boldsymbol{n}]$ in the $SU(2)$ Yang—Mills theory. The Faddeev soliton picture was revived in the Faddeev—Niemi knot model [9], and in the $\boldsymbol{n}$-dependent parametrization of the QCD gauge field [10, 11]. Shabanov [12, 13] used the Cho connectionis to reformulate a Yang—Mills theory as a Abelian gauge theory via a nonlocal change of variables in the space of connections, rather than via a gauge fixing, and stressed an importance of coset variables with related $U(1)$ symmetry.

The Cho—Faddeev—Niemi—Shabanov (CFNS) decomposition for two-color QCD is defined by

$$\boldsymbol{A}_\mu = C_\mu \boldsymbol{n} + g^{-1}[\partial_\mu \boldsymbol{n}, n] + \boldsymbol{X}_\mu, \ \boldsymbol{n} X_\mu = 0,$$

so that the behaviour of all fields is fixed with respect to $\boldsymbol{n}$. The CFNS decomposition was extended to $SU(3)$ [13–16] and it was shown to describe monopoles.

The question of the $\boldsymbol{n}(x)$ field origin was discussed several times [14, 17–20] to the result that $\boldsymbol{n}(x)$ can be treated as a collective dynamic field, which does not introduce additional degrees of freedom. As far as physical implications are concerned, it was shown that for two colors at low energy the CFNS procedure corresponds to a spin-charge separation [11]. This is a new step in the "abelianization" program compared with maximal Abelian gauge approach [21, 22]. In CFNS picture the abelian directions are defined in a gauge invariant way.

The CFNS decomposition as a reformulation of QCD based on change of variables has been recently investigated in detail in series of papers [14, 23, 24] where, in particularly, it was found that the Wilson loop operator and the Polyakov loop operator are constructed out of first two terms in $\boldsymbol{A}_\mu$ alone. Studies of coset spaces appearing in the CFNS picture revealed an existence of vortex solutions [25]. The CFNS approach has been extended to the treatment of defects [26].

However, restriction to the QCD without quarks raises the question, whether such approximation is good. Due to the chiral anomaly [27, 28], at the quantum level the color gauge sector and the color chiral anomalous sector are parts of the total color space. Chiral transformations U of quarks in the Dirac Lagrangian do not change the Yang—Mills field in the gauge sector of the theory, but introduce different decompositions of the Yang—Mills field in terms of arising vector and axial vector components. Among these decompositions we are interested in those, which preserve number of the dynamical degrees of freedom (DOF) and may reveal the topological degrees. It is at the classical level, where such decompositions are equivalent, because the Dirac Lagrangian is chiral invariant. At the quantum level, the quark path integral is changed because of the anomaly, therefore, in this case, an impact of the chiral parametrization on the quark path integral requires a special investigation.

The aim of this paper to consider in details a chiral parametrization (ChP) of the QCD gauge field proposed in papers [29, 30]. We assume that the basic color field $\boldsymbol{m}(x)$ has its origin in the quark chiral field U, as a remnant of U after conditions of consistency are imposed. We introduce the decomposition of the QCD gauge field G_μ via a chiral rotation: the field G_μ in the Dirac Lagrangian is submited to a chiral transformation U initiated by quarks

$$\begin{aligned} \psi &\rightarrow \psi^U = U\psi_L + \psi_R, \\ G_\mu &\rightarrow G_\mu\left(V^U, A^U\right) = V_\mu^U - A_\mu^U, \end{aligned}$$

where $\psi_{L,R}$ denote left (right) quarks, V^U and A^U are correspondingly vector and axial vector components of chirally rotated G_μ. Then the second line would represent the chiral decomposition, if it is possible to find such a special chiral field U_0, when $V_\mu^{U_0}$ is invariant under U_0 and V_μ^U can be calculated as a function $V_\mu^U(G)$, i. e. the condition $\det(\partial V^U/\partial G)= 0$ is satisfied. As a result, this defines $U_0 = \boldsymbol{m}(x)$, $\boldsymbol{m}^2 = 1$, $\operatorname{tr}\boldsymbol{m} = 1$ up to a constant phase. In the case $SU(2)$ we have $V_\mu^{U_0} = \boldsymbol{m}C_\mu + \frac{1}{2}\boldsymbol{m}\partial_\mu\boldsymbol{m}$, while the axial part $A_\mu^{U_0}$ is orthogonal to $\boldsymbol{m}$ in color space and may be identified with $-X_\mu$. Thus, this parametrization is the same as the CFNS one in $SU(2)$. In the $SU(3)$ case, the chiral decomposition selects the $SU(3)/U(2) \simeq CP^2$ color space (minimal symmetry), while the CFNS decomposition admits also the $SU(3)/U(1)^2$ coset space (maximal symmetry). The conclusion of [29, 30] is that the chiral parametrization coincides with the CFNS in the gluon sector, if the unit chiral color vector $\boldsymbol{m}(x)$ is identified with the basic color vector $\boldsymbol{n}(x)$ of CFNS.

It is important that in ChP all properties of decomposition fields follow from chiral transformation rules.

The relation $G_\mu = V_\mu^U - A_\mu^U$ in (1) is invariant under any U-transformations. Among all matrices U there is a chiral $U(1)$ group of matrices $U_\chi = \exp i\boldsymbol{m}\chi$ with $\chi = \text{const}$, which do not change DOF of $V_\mu^{U_0}$ and describe transformations of coset variables. These transformations look dangerous for the chiral decomposition, because in the quark quantum sector they generate the chiral anomaly and the Wess—Zumino—Witten topological action W_{wzw}. In this case W_{wzw} represents quantization condition similar in spirit to the Dirac quantization of monopoles.

Left-Right group $SU(3)_L \times SU(3)_R$ and chiral field. In this section we establish notations for the Left-Right group $SU(3)_L \times SU(3)_R$ and the chiral field. Consider massless fermions in external vector and axial vector fields V_μ, A_μ and the Dirac operator

$$\not{D}(V,A) = i\gamma^\mu\left(\partial_\mu + V_\mu + \gamma_5 A_\mu\right) \tag{1}$$

with antihermitean fields in algebra $SU(3)$, so that $V_\mu = -V_\mu^a it_a$, $A_\mu = -A_\mu^a it_a$, where t_a, $a = 1, 2, \ldots 8$ are generators. The chiral transformation of fermions is given by

$$\psi_L' = \xi_L\psi_L, \psi_R' = \xi_R\psi_R, \psi = \psi_L + \psi_R, \tag{2}$$

where $\xi_L(x)$ and $\xi_R(x)$ are local chiral phase factors of left and right quarks ψ_L and ψ_R, represented by unitary matrices in defining representations of left $SU(3)_L$ and right $SU(3)_R$ subgroups of the chiral group $G_{LR} = SU(3)_L \times SU(3)_R$. For $\psi_L = \frac{1}{2}(1+\gamma_5)\psi$, $\psi_R = \frac{1}{2}(1-\gamma_5)\psi$, generators t_{La} and t_{Ra} of left and right subgroups of G_{LR} can be written as $t_{La} = \frac{1}{4}(1+\gamma_5)\lambda_a, t_{Ra} = \frac{1}{4}(1-\gamma_5)\lambda_a, [t_{La}, t_{Rb}] = 0$, where λ_a, $a = 1, 2, \ldots, 8$ are the Gell-Mann matrices. Then quark left and right chiral phase factors ξ_L, ξ_R arise from application of operators $\hat{\xi}_L = \exp(-it_{La}\omega_{La}), \hat{\xi}_R = \exp(-it_{Ra}\omega_{Ra})$ to left and right quarks ψ_L and ψ_R. Vector gauge transformations g are associated with $t_a = t_{La} + t_{Ra} = \lambda_a/2$, i. e. $g(x)$ has properties of the product $\hat{\xi}_L(x)\hat{\xi}_R(x)$ of identical left and right rotations, $\omega_L = \omega_R = \alpha$. The generator of purely chiral transformations g_5 is $t_{5a} = \gamma_5\lambda_a/2 = t_{La} - t_{Ra}$, thus, g_5 has properties of $\hat{\xi}_L\hat{\xi}_R^+$ for $\omega_L = \omega_R = \Theta$. Infinitesimally, the Dirac operator is transformed according to

$$\delta\not{D} = [i\frac{1}{2}\alpha_a\lambda_a, \not{D}] + \{i\frac{1}{2}\gamma_5\Theta_a\lambda_a, \not{D}\}. \tag{3}$$

Commutation relations for t_a, t_{5a} are given by

$$[t_a, t_b] = if_{abc}t_c, [t_a, t_{5b}] = if_{abc}t_{5c}, [t_{5a}, t_{5b}] = if_{abc}t_c, \tag{4}$$

where f_{abc} are antisymmetrical structure constants of $SU(3)$, we use the normalization $trt_a t_b = 1/2\delta_{ab}$.

Instead of phases ξ_L and ξ_R one can work with the chiral field $U = \xi_R^+ \xi_L$, which describes rotation of only left quark leaving right quark in peace $\psi_L \to \psi'_L = U\psi_L, \psi \to \psi'_R = \psi_R$. The same result can be obtained by the chiral transformation $\psi_L \to \xi_L \psi_L, \psi_R \to \xi_R \psi_R$, followed by a vector gauge transformation with a gauge function ξ_R^+. The usual chiral gauge choice is $\xi_R = \xi_L^+$, then the chiral field is taken as squared left chiral phase:

$$U = \xi_L^2 = \exp i\Pi, \Pi = \Pi_a \lambda_a, \tag{5}$$

where we used the flavor notation Π. This chiral gauge corresponds to the following form of chiral transformation of the Dirac field

$$\psi^U = g_5(U)\psi = (P_L U + P_R)\psi, \ \bar{\psi}^U = \bar{\psi}\bar{g}_5(U) = \bar{\psi}\left(P_R U^+ + P_L\right), \tag{6}$$

which is used in this paper.

To describe U in the case involving monopoles and other defects one can use the basis with the step up/down operators $E_{\pm i}$

$$E_{\pm 1} = \frac{1}{2}(\lambda_1 \pm i\lambda_2), E_{\pm 2} = \frac{1}{2}(\lambda_4 \pm i\lambda_5), E_{\pm 3} = \frac{1}{2}(\lambda_6 \pm \pm i\lambda_7) \tag{7}$$

and diagonal generators

$$H_i = [E_{+i}, E_{-i}], i = 1, 2, 3 \tag{8}$$

with commutation relations

$$[[E_{+i}, E_{-j}], E_{+k}]\delta_{ij}E_{+k} + \delta_{jk}E_{+i}[[E_{+i}, E_{-j}], E_{-k}] = -\delta_{ij}E_{-k} - \delta_{jk}E_{-i} \tag{9}$$

and the $U(1)$ generator of $SU(3) \otimes U(1)$

$$\Lambda_{U(1)} = \frac{1}{4}\sum H_i = \frac{1}{4}\operatorname{diag}(1, 1, -3). \tag{10}$$

Introducing notations for $Z \subset C^3$, normalized $SU(3)$ spinor φ and the CP^2 coordinates $u_\alpha, u_\beta^+, \alpha, \beta = 1, 2$,

$$\begin{aligned} Z &= (z_1, z_2, z_3)^T \subset C^3; \\ Z &= \sqrt{Z^+ Z}(\varphi_1, \varphi_2, \varphi_3)^T = \frac{\sqrt{Z^+ Z}}{\sqrt{1 + u^+ u}}(u_1, u_2, 1)^T \end{aligned}$$

one can parametrize the chiral field U in a form of the $SU(3)$ group element as follows

$$U = \frac{1}{\vartheta}\begin{pmatrix} \Delta & iu \\ iu^+ & 1 \end{pmatrix}, \vartheta = \sqrt{1 + u^+ u},$$

where $u = (u_1, u_2)^T$ and Δ is the 2×2 matrix

$$\Delta_{ij} = \vartheta\delta_{ij} - \frac{u_i u_j^+}{1 + \vartheta}.$$

For applications related to $SO(3)$ monopoles and $SU(3)/SO(3)$ coset, it is convenient to consider $SU(3)$ in the $SO(3)$ basis [31]. $SO(3)$ is the maximal subgroup of $SU(3)$. In this

case it is convenient to use two hermitian combinations: $\hat{N}(x)$ and $\hat{N}^2(x)$, where $\hat{N}(x)$ is built on antisymmetric λ-matrices $\hat{N} = n_k O_k$, $O_k = (\lambda_7, -\lambda_5, \lambda_2)$, $n_k n_k = 1$, $N^3 = N$, while $\hat{N}^2$ contains only symmetric λ's. Then general $SU(3)$-chiral field is

$$U(\alpha,\beta) = \exp i\Pi(\alpha,\beta) = \exp i\left(\hat{N}\alpha + (\frac{1}{2}\{\hat{N}', \hat{N}''\} - \frac{1}{3}\operatorname{tr} N'N'')\beta\right), \tag{11}$$

where $\hat{N}', \hat{N}''$ depend on $SO(3)$ unitvectors n'_k, n''_l. Three unit vectors plus α, β, give altogether 8 parameters of $SU(3)$.

The Dirac Lagrangian remains invariant under chiral transformation of fermions if external fields transform correspondingly

$$\bar{\psi}\, \not{D}(V,A)\,\psi = \bar{\psi}^U D\left(V^U, A^U\right)\psi^U, \tag{12}$$

where

$$V^U = \frac{1}{2}U[\partial U^+ + \{V, U^+\} + [A, U^+]],$$
$$A^U = \frac{1}{2}U[\partial U^+ + [V, U^+] + \{A, U^+\}]. \tag{13}$$

These expressions will be used as the starting point for an introduction of chiral variables into the QCD vector field.

Chiral field as a direction in the color space. Consider the Dirac action $\bar{\psi}\, \not{D}(V,0)\,\psi$ containing only the QCD gauge field, V_μ. There is no axial vector color field, $A = 0$ initially. After the chiral rotation of fermions we get chirally rotated gauge field

$$V^U = \frac{1}{2}U\left[\partial U^+ + \{V, U^+\}\right],\ A^U = \frac{1}{2}U\left[\partial U^+ + [V, U^+]\right]. \tag{14}$$

This is the definition of the (once) chirally rotated field. For subsequent chiral rotations one should use transformation rules (13). Now, V^U is a gauge field

$$V^U \to gV^U g^+ + g\partial g^+, A^U \to gA^U g^+, \tag{15}$$

while A^U transforms homogeneously under gauge transformation $g(x) \subset SU(3)_{L+R}$.

Two simple relations exist for $V^U \mp A^U$, namely

$$V^U_\mu - A^U_\mu = V_\mu,$$
$$V^U_\mu + A^U_\mu = U(\partial_\mu + V_\mu)U^+, \tag{16}$$

which mean that if the chiral field is considered as a regular gauge transformation, then $(V^U \pm A^U)_\mu$-combinations should have the same field strengths $\left(V^U \pm A^U\right)_{\mu\nu}$. With topological U these combinations can describe different situations. The first of these relations may be considered as a chiral parametrization for $V_\mu = V^U_\mu - A^U_\mu$, i. e. as a way to introduce explicitly topological variables of U into the gauge field V_μ. The price of such a parametrization would be the requirement of V_μ-invariance under chiral rotations $U \to U'U$ eliminating superfluous variables. In order to avoid this situation one should restrict the choice of the special chiral field $U(x) = U_0(x)$, to be used for parametrization by the following conditions:

(a) the gauge part V^U_μ of rotated gluon field is invariant under chiral rotation, and

(b) V^U_μ is defined uniquely in terms of V_μ.

Indeed, by introducing the quark color chiral field U we arrive at a system with too many degrees of freedom. To eliminate superfluous variables, one should consider a relation between gluonic fields V_μ and chirally transformed field V_μ^U and find how different parts of these fields can be made from the same material, so that chiral field variables are either fixed, or fully incorporated into gauge field variables.

The invariance condition (a) means that

$$(V_\mu^U)^{U'} = \frac{1}{2}\left[U'^{+}\left(V_\mu^U + A_\mu^U\right)U' + V_\mu^U - A_\mu^U + U'^{+}\partial_\mu U'\right] = V_\mu^U. \tag{17}$$

In terms of the Dirac operator this condition reads

$$\not{D}\left(V_\mu^U, 0\right) = U^{+}\not{D}\left(V_\mu^U, 0\right)U, \tag{18}$$

that implies that the axial field does not appear.

This condition defines a special chiral field

$$U_0 = U_0', U_0^2 = \exp i\zeta, \partial_\mu\zeta = 0,\ U_0 = \boldsymbol{m}\exp i\zeta/2, \tag{19}$$

where $\boldsymbol{m}$ is a hermitian $SU(3)$ matrix with $\boldsymbol{m}^2 = \boldsymbol{I}$ describing a direction in the color space and ζ is a real number. With the chiral field U_0 an axial vector field $A_\mu^{U_0}\left(V_\mu^{U_0}\right)$ calculated with $V_\mu^{U_0}$ instead of V_μ will disappear

$$A_\mu^{U_0}\left(V_\mu^{U_0}\right) = \frac{1}{2}[U_0^{+}(\partial_\mu + V_\mu^{U_0})U_0 - V_\mu^{U_0}] = 0. \tag{20}$$

Note that Eqs following from the condition (a) are independent of the chiral color group.

Consider the condition (b) This can be done by studying the $(V_\mu^U \to V_\mu)$-determinant. We have the following relation between 8×8 matrices of gluonic field V_μ and chirally rotated field V_μ^U in adjoint representation

$$(V_\mu^U)_{ab} = \frac{1}{2}\left(1 + R\left(U\right)\right)_{ab}V_{\mu b} + \frac{i}{g}\frac{1}{2}(U\partial_\mu U^{+})_{ab}, R_{ab}\left(U\right) = \frac{1}{2}\operatorname{tr}\left(\lambda_a U\lambda_b U^{+}\right), \tag{21}$$

where the chiral field $U = \xi_L^2 = \exp i\Theta, \Theta = \lambda_a\Theta_a$, is defined in the chiral gauge $\xi_L = \xi_R^{+}$. Here $\lambda_a, a = 1, 2..8$, are the Gell-Mann matrices.

In order to calculate the determinant $\det\frac{1}{2}\left(1 + R\left(U\right)\right)$, it is sufficient to consider the parametrization (11) for U in the case $N' = N'' = N$. We get

$$U \to U\left(\hat{N}, \alpha, \beta\right) = \exp\left(-\frac{2}{3}i\beta\right)\left[1 + i\hat{N}e^{i\beta}\sin\alpha + \hat{N}^2\left(e^{i\beta}\cos\alpha - 1\right)\right]. \tag{22}$$

When eigenvalues of $\hat{N}$ are placed as $\operatorname{diag}(1, -1, 0)$, we see that $\alpha = \Theta_3, \beta = \Theta_8\sqrt{3}$ and

$$\det\frac{1}{2}\left(1 + R\left(U\right)\right) = \frac{1}{2}\left(1 + \cos 2\alpha\right)\frac{1}{2}\left(1 + \cos\left(\alpha + \beta\right)\right)\frac{1}{2}\left(1 + \cos\left(\alpha - \beta\right)\right). \tag{23}$$

This result can be easily checked in (isospin I_3, hypercharge Y) basis, where the diagonal elements of $\mathrm{R}(U) = \exp i\left(I_3 2\alpha + Y 2\beta\right)$, as of the adjoint representation of U, are nothing, but those of octet: pions ($I = 1, Y = 0$), K-mesons ($I = 1/2, Y = 1/2$), $\bar{\mathrm{K}}$-mesons ($I = 1/2, Y = -1/2$), σ-meson ($I = 0, Y = 0$).

This determinant is invariant under reflection $(\alpha,\beta)\to(-\alpha,-\beta)$. It disappears for values (α,β) equal to $(\pi/2,0)$, $(\pi/2,\pm\pi/2)$ and $(0,\pi)$ characterizing singularity surfaces in chiral color space (i. e. $\gamma_5\Theta$). The first of these sets, $(\pi/2,0)$, corresponds to $SO(3)$ subgroup with one zero factor in det, when $U(N,\pi/2,0)=\exp i\hat{N}\alpha=1+i\hat{N}-\hat{N}^2$.

For $SU(3)$ we are interested in two coinciding zero factors of det related to two simple roots of $SU(3)$. Together with a singularity set related to a complex root, we can define three color chiral fields $U(\hat{N},\alpha,\beta)$ for $SU(3)$. For the pair $(\alpha,\beta)=(0,\pi)$ we get the chiral field

$$U\left(\hat{N},0.\pi\right)=(1-2\hat{N}^2)\exp(-i\frac{1}{3}\pi)=m_1\exp(-i\frac{1}{3}\pi). \tag{24}$$

For $(\alpha,\beta)=(\pi/2,\pm\pi/2)$ we have

$$U\left(\hat{N},\pi/2,\pi/2\right)=\left(1-N-N^2\right)\exp\left(-i\frac{\pi}{3}\right)$$

$$=m_2\exp\left(-i\frac{\pi}{3}\right)U\left(\hat{N},\pi/2,-\pi/2\right)=\left(1+N-N^2\right)\exp\left(+i\frac{\pi}{3}\right). \tag{25}$$

In all these cases m_k are normalized hermitian 3×3 matrices in color space: $m^2=\boldsymbol{I}$. A product of two m's is equal to the third m up to a constant phase. With the diagonal $\hat{N}=\mathrm{diag}\,(1,-1,0)$, the diagonal forms of m_k are given by

$$m_1^0=\mathrm{diag}\,(-1,-1,1)\,,\ m_2^0=\mathrm{diag}\,(-1,1,1)\,,\ m_3^0=\mathrm{diag}\,(1,-1,1)\,. \tag{26}$$

These matrrices have simple meaning: they are related to 2π rotation of diagonal operators in $SU(2)$ subgroups of $SU(3)$

$$m_1^0=-\exp iI_3 2\pi,\ m_2^0=-\exp iU_3 2\pi,\ m_3^0=\exp iV_3 2\pi, \tag{27}$$

where $I_3=\frac{1}{2}\lambda_3$ for isospin subgroup, $U_3=-\frac{1}{4}\lambda_3+\frac{\sqrt{3}}{4}\lambda_8$, for U-spin subgroup, $V_3=\frac{1}{4}+\frac{\sqrt{3}}{4}\lambda_8$ for V-spin subgroup. From the view point of the Cartan—Weyl basis these matrices correspond to rotation around simple roots (m_1^0 and m_2^0) and around the composite root (m_3^0). Matrices m_k^0 are unitary equivalent.

Thus, behavior of the $(V_\mu^U\to V_\mu)$ determinant $\det\frac{1}{2}(1+R)$ shows, how to restrict chiral color space in defining the chiral field U_0. Chiral invariant regions Ω are given by sets $(\alpha,\beta)=(0,\pi)\,,(\pi/2,\pm\pi/2)$, where the chiral field $U=m_k$ is represented correspondingly by one of 3×3 unit matrices m_k, $k=1,2,3$ up to a constant phase. Our choice is

$$U_0=-Sm_1^0S^+\equiv Sm^0S^+=\boldsymbol{m},$$

$$m^0=\mathrm{diag}\,(1,1,-1)=\frac{2}{\sqrt{3}}\lambda_8+\frac{1}{3}=\exp i\pi\left(Y-\frac{1}{3}\right), \tag{28}$$

where S is a unitary $SU(3)$ transformation and Y is the color hypercharge. Independence of m_0 under global group $U(1)\notin SU(3)$ is essential. It is introduced as a chiral field U_0 up to a phase. Also, it is a group element of $SU(3)$ up to a phase and in this quality participates in the CP^2 involution operation (see para 4). It is the generator of $U(1)$ invariance group of CFNS decomposition (para 5). All field quantities constructed from the color field $\boldsymbol{m}$ do not depend on this global phase.

The chiral field $U_0=\boldsymbol{m}$ is an orbit of $SU(3)$ through the hypercharge $Y=\lambda_8/\sqrt{3}$. Then S is defined up to right multiplication $S\to Sh$, where $h\in U(2)$, i. e. S is in the coset

$$SU(3)/U(2)=CP^2.$$

Such manifold is known as the complex projective space CP^2. Under the gauge transformation $g \in SU(3)_{R+L}$ we have $\boldsymbol{m} \to g\boldsymbol{m}g^+$. It means that the direction $\boldsymbol{m}$ is changed by gauge transformations defined in terms of CP^2 variables. By a special choice of the gauge ("unitary gauge") this direction may be made coinciding with $\boldsymbol{m}_0 = 2Y + 1/3$. Thus, CP^2 variables describe orientational degrees of freedom contained in the unit color vector $\boldsymbol{m}$. As we shall see below, this quantity brings quark chiral topological defects into the gluon field.

Structure of $\boldsymbol{m}$ rotated fields: $\boldsymbol{m}$-even and $\boldsymbol{m}$-odd fields. The rules (13) of chiral transformation for the gauge field G_μ (in absence of an axial vector field in initial state) by the special chiral field

$$U_0 = \boldsymbol{m},\ \boldsymbol{m}^2 = 1,\ \operatorname{tr}\boldsymbol{m} = 1$$

define essential features of rotated fields structure. Let us rewrite the rules (13) in the form

$$G_\mu^m = \frac{1}{2}\left(m\left\{G_\mu, m\right\} + m\partial_\mu m\right),$$

$$A_\mu^m = \frac{1}{2}\left(m\left[G_\mu, m\right] + m\partial_\mu m\right) = \frac{1}{2} m D_\mu(G)\, m = \frac{1}{4}\left[m, D_\mu(G)\, m\right], \tag{29}$$

where $D_\mu(G)\, m = \partial_\mu m + \left[G_\mu, m\right]$ and we used identity $D_\mu(G)\, m^2 = 0$. It follows from (11), that with respect to m the gauge field G may be subdivided into commuting $G_\parallel$ and anticommuting $G_\perp$ parts

$$G = G_\parallel + G_\perp,$$

$$\left[G_\parallel, m\right] = 0, \{G_\perp, m\} = 0. \tag{30}$$

Only $G_\parallel$ contributes to G^m and only $G_\perp$ contributes to A^m

$$G_{\parallel\mu}^m = G_{\parallel\mu} + \frac{1}{2} m\partial_\mu m$$

$$A_\mu^m = G_{\perp\mu} + \frac{1}{2} m\partial_\mu m. \tag{31}$$

From $m^2 = 1$ follows that $\partial_\mu m$ anticommutes with m, hence we have the relations

$$\left\{G_\mu^m, m\right\} = \left\{G_\mu, m\right\}; \tag{32a}$$

$$\left[G_\mu^m, m\right] = -\partial_\mu m; \tag{32b}$$

$$\left\{A_\mu, m\right\} = 0; \tag{32c}$$

$$\left[A_\mu^m, m\right] = 2A_\mu^m m; \tag{32d}$$

$$\left[m, D_\mu(G^m)\, m\right] = 0; \tag{32e}$$

$$\left[m, \left[\partial_\nu m, \partial_\mu m\right]\right] = 0. \tag{32f}$$

Within the part $G_\parallel$ we distinguish an abelian field mC_μ and an $U(2)$ field Q_μ both commuting with m

$$G_\parallel = mC_\mu + Q_\mu,\ \left[Q_\mu, m\right] = 0,\ C_\mu = \operatorname{tr} mG_\mu, \tag{33}$$

so that

$$G_\mu^m = G_{\parallel\mu} + \frac{1}{2} m\partial_\mu m = V_\mu^\Omega + Q_\mu, \tag{34}$$

where we introduced a special notation

$$V_\mu^\Omega = mC_\mu + \frac{1}{2} m\partial_\mu m, \tag{35}$$

for the CP^2 field satisfying the invariance condition $m \not{D}\left(V_\mu^\Omega, 0\right) m = \not{D}\left(V_\mu^\Omega, 0\right)$. The matrix $\boldsymbol{m}$ is covariantly constant

$$D_\mu\left(V_\mu^\Omega\right) m = 0. \tag{36}$$

To complete the set of relations for m-rotated fields we remind two identities

$$G_\mu^m - A_\mu^m = G_\mu$$

$$G_\mu^m + A_\mu^m = m\, G_\mu\, m + m\partial_\mu m.$$

Transformations of the Dirac operator $\not{D}(G, 0)$ due to consequent chiral rotations by $U_0 = m$

$$\psi \to \psi^m = (P_L m + P_R)\psi,\ \bar{\psi} \to \bar{\psi}^m = \bar{\psi}(P_R m + P_L),\ P_{L,R} = \frac{1}{2}(1 \pm \gamma_5) \tag{37}$$

are given schematically by

$$\not{D}(G, 0) \to \bar{\psi}^m\, \not{D}(G, 0)\, \psi^m = \not{D}(G^m, A^m) \to$$

$$\bar{\psi}^m\, \not{D}(G^m, A^m)\, \psi^m = \not{D}(G, 0) = \not{D}(G^m - A^m, 0). \tag{38}$$

In the simplest case when the gauge field is absent, $G_\mu = 0$, we get

$$\bar{\psi}^m\, \not{D}(0, 0)\, \psi^m = \not{D}\left(\frac{1}{2} m\partial_\mu m, \frac{1}{2} m\partial_\mu m\right) = \not{D}\left(\Gamma^0, \Gamma^0\right), \tag{39}$$

where

$$\Gamma_\mu^0 = \frac{1}{2} m\partial_\mu m = \frac{1}{4}\left[m, \partial_\mu m\right] \tag{40}$$

is a m-invariant connection.

This connection (the Cho connection) Γ_μ^0 is an important element of the decomposition of Yang—Mills field proposed by [8, 9, 14] or the CFN-decomposition.

Chiral paramertrization. The chiral parametrization starts when we introduce into this line of identities the field G_μ^m including the connection Γ_μ^0 and represent the gauge field as

$$G_\mu = \Gamma_\mu^0 + mC_\mu + Q_\mu + X_\mu \tag{41}$$

and consider the parametrisation of X_μ in accordance with its properties as the axial vector field $\left(-A_\mu^m\right)$. According to (30) X_μ should be equal to $G_\perp$: while Q_μ belongs to $G_\parallel$

$$\left\{X_\mu, m\right\} = 0,\ \left[Q_\mu, m\right] = 0.$$

It is easy to verify that the Dirac operator has a property

$$m\, \not{D}\left(V_\mu^\Omega + Q_\mu + X_\mu, 0\right) m = \not{D}\left(V_\mu^\Omega + Q_\mu, -X_\mu\right), \tag{42}$$

and we can use for $\left(-X_\mu\right)$ all relations derived for A^m.

Let us construct $X_\mu = G_{\perp\mu} = -A^m_\mu$ as a field anticommuting with $U_0 = m$. Note that this property does not depend on a particular color group. Within the algebra $SU(3)$ the matrix $m_0 = \mathrm{diag}\,(1,1,-1)$ anticommutes with $\lambda_t, t = 4,5,6,7$. Hence, a gauge transformed matrix $m = Sm_0S^+$ anticommutes with $S\lambda_t S^+$. Then, X_μ can be represented as

$$X_\mu = S\lambda_t x_{t\mu} S^+, t = 4,5,6,7, \tag{43}$$

with color invariant fields $x_{t\mu}$. Another example of matrices anticommuting with m provide derivatives $\partial_\mu m$ and $m\partial_\mu m$, which we use to construct a contribution Y_μ to A^U_μ

$$Y_\mu = \varphi\partial_\mu m + i\chi m\partial_\mu m, \tag{44}$$

where φ and χ are colorless functions. Analogous terms with derivatives exist in the two-color decomposition [9]. However, anticommutativity with the matrix m is not sufficient to define $G_\perp = X_\mu + Y_\mu$. We should count number of degrees of freedom contained in the chiral parametrization and consider gauge fixing. Thus, the relation $G_\perp = X_\mu + Y_\mu$ gives us only a choice of eligible terms which can be different for different applications.

Thus, as it follows from expressions for the QCD vector field ('gluons') V_μ there are two distinct sectors in the chiral decomposition of $V_\mu = V^\Omega_\mu + X_\mu$:

(a) the CP^2-sector with the dynamical abelian field C_μ. The space CP^2 enters the scene with the chiral field $U = m = Sm_0S^+$, as the $SU(3)$ orbit through m_0. The matrix m is the main element, which defines the vector field in the sector $\left(V^\Omega_\mu\right)_{CP^2} = mC_\mu + \frac{1}{2}m\partial_\mu m$, including the direction of an abelian field C_μ in the $SU(3)$ space. The axial component $\hat{\boldsymbol{A}}_\mu$ anticommutes with m.

(b) an $U(2)$ sector with the field $\left(V_\mu\right)_{U(2)} = Q_\mu$; the chiral field $U = m$ commutes with Q.

Transformation properties of all field operators follow directly from those (13) of the Left-Right group. Let us recapitulate them in detail. After chiral field is fixed at $U = U_0 = m$, it is only the standard gauge transformation $g(x) \in U(3)$ common for gluons and quarks (i. e. for G_μ and m) which is left for all variables. With $g(x) \simeq 1 + \omega$ we get

$$\begin{gathered}\delta G_\mu = -D_\mu(G)\,\omega,\ \delta m = [\omega, m],\ \delta C_\mu = -m\partial_\mu\omega,\ \delta A^m_\mu = \left[\omega, A^m_\mu\right],\\ \delta\Gamma_\mu = \left[\omega, \Gamma_\mu\right] - \frac{1}{2}\left(m\partial_\mu\omega m - \partial_\mu\omega\right),\\ \delta Q_\mu = \left[\omega, Q_\mu\right] + \frac{1}{2}\left(m\partial_\mu\omega m + \partial_\mu\omega\right).\end{gathered} \tag{45}$$

However, when we go over from matrix $\boldsymbol{m} = Sm_0S^+$ to its building blocks in terms of m. e. of S, we find a hidden $U(1)$ group related to local transformation $S_{\alpha 3} \to S_{\alpha 3}\exp i\chi$ leaving invariant the projector $P = \frac{1}{2}(1 - \boldsymbol{m}) = S\,\mathrm{diag}\,(0,0,1)\,S^+$. This $U(1)$ group is not a subgroup of the gauge $SU(3)_{L+R}$ of QCD. It comes as a hidden signature of quarks. Its existence has been shown by Shabanov [13].

CP^2 sector in the chiral parametrization. The space CP^2 in QCD emerged in the parametrization of the vector field, when the chiral color direction m, as a remnant of the quark chiral field, was introduced into the gauge field. Being by origin a quark variable the field $m(x)$ can be considered also as a gluonic variable.

m is a 3×3 hermitean matrix $m = m_a\lambda_a$; $a = 0,1,2,\ldots 8$, with the properties $m^2 = 1$, $\mathrm{tr}\, m = 1$. The matrix m can be expressed as

$$m = Sm_0S^+,\ m_0 = \mathrm{diag}\,(1,1,-1) = \frac{2}{\sqrt{3}}\lambda_8 + \frac{1}{3} = 2Y + \frac{1}{3},$$

$$S \in SU(3)\,,\ SS^{+} = S^{+}S = 1,$$

m is an orbit of $SU(3)$ through hypercharge $Y = \frac{1}{\sqrt{3}}\lambda_8$ and describes the complex projective space CP^2 in terms of matrix elements of S. Due to unitarity of S there are different ways of description of CP^2:

(a) Representation of S in terms of the tangent bundle T of λ_8, when

$$S = \exp iT,\ T = \hat{\alpha}\omega,\ \hat{\alpha} = \lambda_t \alpha_t,\ t = 4,5,6,7;\ T^{+} = T, \tag{46}$$

α is given by $\alpha_{3\alpha} = \xi_{\alpha}^{+}$; $\alpha_{\alpha 3} = \xi_{\alpha}$, $\xi^{+}\xi = 1, \alpha_{\alpha\beta} = 0$, so that ξ, ξ^{+}, ω represent four independent variables of CP^2.

$$\alpha = \begin{pmatrix} 0 & 0 & \xi_1 \\ 0 & 0 & \xi_2 \\ \xi_1^{+} & \xi_2^{+} & 0 \end{pmatrix}, \tag{47}$$

where ξ_{α}, $\alpha = 1,2$, is normalized according to $\xi^{+}\xi = 1$, and ξ, ξ^{+}, ω represent four independent variables of CP^2. Due to relation $\alpha^3 = \alpha$ we have the closed expression

$$S = 1 + i\hat{\alpha}\sin\omega + \hat{\alpha}^2(\cos\omega - 1)\,, \hat{\alpha}_{\alpha\beta}^2 = \xi_{\alpha}\xi_{\beta}^{+}, \alpha_{33}^2 = 1, \alpha_{\alpha 3}^2 = 0. \tag{48}$$

The chiral matrix $\boldsymbol{m}$ anticommutes with $\hat{\alpha}, \{\hat{\alpha}, m\} = 0$, as a result

$$\boldsymbol{m} = S^2\,\boldsymbol{m}_0. \tag{49}$$

The hidden $U(1)$ symmetry is described by transformation $\xi_{\alpha} \to e^{i\chi}\xi_{\alpha}$ which leaves α^2 invariant.

$SU(3)$ matrices Q commuting with m can be built on S-transforms of $Q^0 = \lambda_q Q_q^0$, $q = 1,2,3,8$, namely $Q = SQ^0S^{+}$.

(b) Representation of m in terms of projection operators.

One can consider CP^2 as a space of hermitean 3×3 projection operators

$$P = P_a\lambda_a,\ a = 0,1,2...8,\ P^2 = P,\ trP = 1, \lambda_0 = \boldsymbol{I} = \mathrm{diag}(1,1,1) \tag{50}$$

To arrive at CP^2 space, we can start with $P^0 = \frac{1}{2}(1 - m_0) = \mathrm{diag}(0,0,1)$ and construct $P = SP^0S^{+} = \frac{1}{2}(1 - m)$, where S is a general unitary $SU(3)$ matrix (which always can be rewritten in a form containing only the tangent bundle T). If we consider a given expression for a projection operator, it is necessary to check defining relations $\mathrm{tr}\,P = 1, P^2 = P$, which lead to the following conditions on P_a

$$P_0 = \frac{1}{3}, P_aP_a = \frac{1}{3}; d_{abc}P_aP_b = \frac{P_c}{3}, \tag{51}$$

where d_{abc} are symmetric structure constants $d_{abc} = \frac{1}{4}\,\mathrm{tr}\,(\lambda_a\{\lambda_b, \lambda_c\})$ with $a,b,c \neq 0$. This definition describes CP^2 as a submanifold of the 8-dimensional Euclidean space R^8, or of R^9 with fixed P_0.

Projection operators P can be realized by means of normalized $SU(3)$ spinors $\varphi = (\varphi_1, \varphi_2, \varphi_3)^T$ considered as unit vectors $\mid \varphi \succ$ in C^3 modulo the phase

$$P_{ij} = \varphi_i\varphi_j^{+}, m_{ij} = -2P_{ij} + \delta_{ij}, \tag{52}$$

assuming $\varphi \neq 0$. $SU(3)$ spinors φ and φ^{+} represent matrix elements S_{k3} and S_{3j}^{+} of unitary S in definition $m = Sm_0S^{+}$. This representation should be remembered in understanding expressions $P = \varphi\varphi^{+}$ and $[P, \varphi] = \varphi$.

In CP^2 space points φ and $\varphi' = \varphi e^{i\alpha}$ connected by an $U(1)$ transformation are equivalent.

A point of CP^2 in terms of φ is

$$X_a(\varphi) = \varphi^+ \lambda_a \varphi, \tag{53}$$

$SU(3)$ spinors φ and φ^+ anticommute with m

$$\{m, \varphi\} = \{1 - 2P, \varphi\} = 2\varphi - 2\left\{\varphi\varphi^+, \varphi\right\} = 0.$$

In terms of unitary matrix S introduced by the relation $m = Sm_0S^+$ we have $\varphi_j = S_{j3}$, $j = \alpha, 3$, so that $\varphi^T = (i\xi_1 \sin\omega, i\xi_2 \sin\omega, \cos\omega)$.

Normalized $SU(3)$ spinors are not convenient in treatment of nontrivial topological configuratiions. We can parametrize the CP^2 space by a set of coordinates $z = (z_1, z_2, z_3)$, which is invariant to multiplication by a complex scalar $z \to \lambda z$. Then a projection operator can be written as

$$P_{ij} = \frac{z_i z_j^+}{z_k^+ z_k}. \tag{54}$$

Coordinate z can be parametrized by four angles

$$z = \left(\sin\theta\cos\phi e^{i\beta}, \sin\theta\sin\phi e^{i\gamma}, \cos\theta\right), \tag{55}$$

where $0 \leqslant \theta, \phi \leqslant \pi/2; 0 \leqslant \beta, \gamma \leqslant 2\pi$.

QCD vector field and Effective gluonic lagrangian. Let us consider the Lagrangian for the gluon field

$$L = -\frac{1}{2g^2}\,\mathrm{tr}\, G_{\mu\nu}^2,$$

and express it in CP^2 variables.

QCD vector field in CP^2 sector. We consider the gauge field $V_\mu^\Omega = mC_\mu + \frac{1}{2}m\partial_\mu m$ (35) (the CNFS connection) and corresponding field strength

$$V_{\mu\nu}^\Omega = mC_{\mu\nu} + \frac{1}{4}\left[\partial_\mu m, \partial_\nu m\right]. \tag{56}$$

We use the representation of m in terms of projection operators (52). After straitforward calculations we get for the CNFS connection in the CP^2 variables

$$\frac{1}{4}\left[m, \partial_\mu m\right] = \left[P, \partial_\mu P\right] = 2\left(b_\mu P - B_\mu\right) \tag{57}$$

where the magnetic (classical) vector b_μ and magnetic matrix B_μ are given by

$$b_\mu = \frac{1}{2}\left(\varphi_i^+ \partial_\mu \varphi_i - \partial_\mu \varphi_i^+ \varphi_i\right), \tag{58}$$

$$(B_\mu)_{ij} = \frac{1}{2}\left(\partial_\mu \varphi_i \varphi_j^+ - \varphi_i \partial_\mu \varphi_j^+\right). \tag{59}$$

When the relation $\varphi_i^+ \varphi_i = 1$ is used, we have $b_\mu = \varphi_i^+ \partial_\mu \varphi_i$.

To avoid overloading the formula, we omit the Latin indices in the further presentation of the material.

Finally, the gauge field in CP^2 variables is

$$V_\mu^\Omega = \left(1 - 2\varphi\varphi^+\right) C_\mu + 2\left(b_\mu\varphi\varphi^+ - B_\mu\right) = C_\mu + \left(\varphi\bar{D}_\mu^{bC}\varphi^+ - D_\mu^{bC}\varphi\cdot\varphi^+\right) = C_\mu + V_\mu^\Phi, \quad (60)$$

where

$$D_\mu^{bC} = \left(\partial_\mu - b_\mu + C_\mu\right), \bar{D}_\mu^{bC} = \left(\partial_\mu + b_\mu - C_\mu\right).$$

We introduce notation

$$V_\mu^\Phi = \left(\varphi\bar{D}_\mu^{bC}\varphi^+ - D_\mu^{bC}\varphi\cdot\varphi^+\right). \quad (61)$$

Note the relations

$$\bar{D}_\mu^{bC}\varphi^+\cdot\varphi \quad = \quad -C_\mu\varphi^+\varphi, \ \varphi^+ D_\mu^{bC}\varphi = C_\mu\varphi^+\varphi.$$

The CFN field strength $\frac{1}{4}\left[\partial_\mu m, \partial_\nu m\right]$ in Eq. (56) in CP^2 variables looks complicated

$$\frac{1}{4}\left[\partial_\mu m, \partial_\nu m\right] = Pb_{\mu\nu} + 2\hat{B}_{\mu\nu}, \quad (62)$$

where

$$b_{\mu\nu} = \partial_\mu\varphi^+\partial_\nu\varphi - \partial_\nu\varphi^+\partial_\mu\varphi \quad (63)$$

$$\hat{B}_{\mu\nu} = \frac{1}{2}\left(D_\mu^b\varphi\bar{D}_\nu^b\varphi^+ - D_\nu^b\varphi\bar{D}_\mu^b\varphi^+\right). \quad (64)$$

We use notations $D_\mu^b = \partial_\mu - b_\mu, \ \bar{D}_\mu^b = \partial_\mu + b_\mu$.

It is convenient to introduce new matrixes $B_{\mu\nu}$ and $R_{\mu\nu}$

$$\hat{B}_{\mu\nu} = B_{\mu\nu} - \frac{1}{2}R_{\mu\nu}. \quad (65)$$

The matrixes $B_{\mu\nu}$ and $R_{\mu\nu}$ are given by expressions

$$B_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\varphi\partial_\nu\varphi^+ - \partial_\nu\varphi\partial_\mu\varphi^+\right) \quad (66)$$

$$R_{\mu\nu} = \left(b_\mu\partial_\nu\left(\varphi\varphi^+\right) - b_\nu\partial_\mu\left(\varphi\varphi^+\right)\right) \quad (67)$$

with properties

$$\mathrm{tr}\, B_{\mu\nu}^2 = b_{\mu\nu}^2, \ \left[B_\mu, B_\nu\right] = \frac{1}{2}\left(b_{\mu\nu} + B_{\mu\nu}\right), \ \left\{\varphi\varphi^+, B_{\mu\nu}\right\} = 0, \ \left\{R_{\mu\nu}, \varphi\varphi^+\right\} = 0. \quad (68)$$

The abelian part

$$C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \quad (69)$$

can be picked out in the gauge field strength (56)

$$V_{\mu\nu}^\Omega = C_{\mu\nu} + V_{\mu\nu}^\Phi, \quad (70)$$

where one can express $V_{\mu_\nu}^\Phi$ in terms of matrixes $B_{\mu\nu}$ and $R_{\mu\nu}$

$$V_{\mu\nu}^\Phi = \varphi\varphi^+\left(-2C_{\mu\nu} + b_{\mu\nu}\right) + 2B_{\mu\nu} - R_{\mu\nu}. \quad (71)$$

QCD vector field in $U(2)$-sector. The gauge field in $U(2)$ sector is $Q_\mu(x)$. In order to deal with Q_μ and φ, φ^+, we replace $Q_\mu \to \hat{Q}_\mu = P_Q Q_\mu P_Q$, introducing the projector $P_Q = (1 - \varphi\varphi^+)$ onto the subgroup $U(2)$. It follows

$$\hat{Q}_\mu \varphi = 0,\ \varphi^+ \hat{Q}_\mu = 0,\ \hat{Q}_\nu \partial_\mu \varphi = -\partial_\mu \hat{Q}_\nu \varphi$$

$$\left[\partial_\mu m, Q_\nu\right] = \left[\partial_\mu \varphi\varphi^+ + \varphi\partial_\mu\varphi^+, Q_\nu\right] = -\left[\varphi\varphi^+, \partial_\mu Q_\nu\right]$$

The field $Q_\mu(x)$ does not commute with the connection $\frac{1}{2} m \partial_\mu m$

$$\left[\frac{1}{2} m\partial_\mu m, Q_\nu\right] = -\frac{1}{2} m\left[m, \partial_\mu Q_\nu\right] = \frac{1}{2}\left\{\varphi\varphi^+, \partial_\mu Q_\nu\right\}.$$

Field strength $V_{\mu\nu}^{U_2}$ in the U(2) sector consists out of $Q_{\mu\nu} = \partial_\mu Q_\nu - \partial_\nu Q_\mu$ and mixed term $\left[V_\mu^\Omega, Q_\nu\right] + \left[Q_\mu, V_\nu^\Omega\right]$

$$V_{\mu\nu}^{U_2} = Q_{\mu\nu} + \left[V_\mu^\Omega, Q_\nu\right] + \left[Q_\mu, V_\nu^\Omega\right] = Q_{\mu\nu} + S_{\mu\nu}, \tag{72}$$

where

$$S_{\mu\nu} = \frac{1}{2}\left\{\varphi\varphi^+, Q_{\mu\nu}\right\}.$$

Axial vector A_μ^m arising after chiral rotation $U = m$ from G_μ. Consider the axial vector part $A_\mu = A_\mu^m$ in the decomposition $G_\mu = V_\mu^\Omega - A_\mu$. The field A_μ anticommutes with m, hence, taking account of $\{m, \varphi\} = 0$ one can write the decomposition of A_μ as

$$\hat{A}_\mu = \hat{K}_\mu \varphi^+ + \varphi K_\mu^+,\ \operatorname{tr} A_\mu = 0,\ \varphi^+ K_\mu = 0,\ K_\mu^+ \varphi = 0. \tag{73}$$

We remind that $\varphi \neq 0$. These terms correspond to tangential structures for m in $SU(3)$ $S\eta_t S^+$ and $S\eta_t^+ S^+$, $t = 1, 2$; in combination $\eta_1 K^{(1)} + \eta_2 K^{(2)}$, where $\eta_1 = \frac{1}{2}(\lambda_4 + i\lambda_5)$, $\eta_2 = \frac{1}{2}(\lambda_6 + i\lambda_7)$

$$\hat{A}_\mu = \sum_t \left(S\eta_t K_\mu^{(t)} S^+ + S\eta_t^+ \bar{K}_\mu^{(t)} S^+\right). \tag{74}$$

Indeed, considering this expression in detail we have with $S_{3k}^+ = \varphi_k^+, S_{k3} = \varphi_k$

$$\left(S\eta_t K_\mu^{(t)} S^+\right)_{kk'} = S_{k't} K_\mu^{(t)} S_{3k}^+ = \hat{K}_{\mu k'} \varphi_k^+,$$

$$\left(S\eta_t^+ \bar{K}_\mu^{(t)} S^+\right)_{kk'} = S_{k3} \bar{K}_\mu^{(t)} S_{tk'}^+ = \varphi_k \hat{K}_{\mu k'}^+$$

— decomposition (73). Contribution of A_μ to $V_{\mu\nu}$ is given by

$$\left[\hat{A}_\mu, \hat{A}_\nu\right] = \left(K_\mu K_\nu^+ - K_\mu K_\nu^+\right) - \varphi\varphi^+ tr\left(K_\mu K_\nu^+ - K_\mu K_\nu^+\right). \tag{75}$$

This term commutes with m

$$\left[\left[\hat{A}_\mu, \hat{A}_\nu\right], m\right] = 0.$$

Consider $\hat{A}_{\mu\nu} = D_\mu \hat{A}_\nu - D_\nu \hat{A}_\mu$, where

$$D_\mu \hat{A}_\nu = \left(\partial_\mu \hat{A}_\nu + \left[V_\mu^\Omega + \hat{Q}_\mu, \hat{A}_\nu\right]\right). \tag{76}$$

We get directly

$$\left[V_\mu^\Omega, \hat{A}_\nu\right] = -\left(D_\mu^{bC}\varphi\hat{K}_\nu^+ + \hat{K}_\nu\bar{D}_\mu^{bC}\varphi^+\right) \tag{77}$$

$$\left[\hat{Q}_\mu, A_\nu\right] = \hat{Q}_\mu\hat{K}_\nu\varphi^+ - \varphi\hat{K}_\nu^+\hat{Q}_\mu. \tag{78}$$

Finally, $D_\mu\hat{A}_\nu$ does not contain derivatives of φ and φ^+

$$D_\mu\hat{A}_\nu = \left(\nabla_\mu\hat{K}_\nu\right)\varphi^+ + \varphi\bar{\nabla}_\mu\hat{K}_\nu^+\,, \tag{79}$$

where

$$\nabla_\mu\hat{K}_\nu = \left(\partial_\mu + \hat{Q}_\mu + b_\mu - 2C_\mu\right)\hat{K}_\nu\ ;\ \bar{\nabla}_\mu\hat{K}_\nu^+ = \left(\partial_\mu - \hat{Q}_\mu - b_\mu + 2C_\mu\right)\hat{K}_\nu^+. \tag{80}$$

Thus, the term $\hat{A}_{\mu\nu}$ in $G_{\mu\nu}$ anticommutes with m

$$\left\{\hat{A}_{\mu\nu}, m\right\} = 0.$$

The field strength $G_{\mu\nu}$. The decomposition of the field strength is

$$G_{\mu\nu} = V_{\mu\nu}^\Omega + V_{\mu\nu}^{U_2} + \left[\hat{A}_\mu, \hat{A}_\nu\right] - \left(D_\mu\hat{A}_\nu - D_\nu\hat{A}_\mu\right), \tag{81}$$

where we remind

$$V_{\mu\nu}^\Omega = C_{\mu\nu} + V_{\mu\nu}^\Phi$$

$$V_{\mu\nu}^\Phi = \varphi\varphi^+\left(-2C_{\mu\nu} + b_{\mu\nu}\right) + D_\mu^b\varphi\bar{D}_\nu^b\varphi^+ - D_\nu^b\varphi\bar{D}_\mu^b\varphi^+$$

$$V_{\mu\nu}^{U_2} = Q_{\mu\nu} + S_{\mu\nu} = Q_{\mu\nu} + \frac{1}{2}\left\{\varphi\varphi^+, Q_{\mu\nu}\right\}, Q_{\mu\nu} = \partial_\mu Q_\nu - \partial_\nu Q_\mu$$

$$\left[\hat{A}_\mu, \hat{A}_\nu\right] = \left(K_\mu K_\nu^+ - K_\mu^+ K_\nu\right) - \varphi\varphi^+ tr\left(K_\mu K_\nu^+ - K_\mu^+ K_\nu\right)$$

$$\left(D_\mu\hat{A}_\nu - D_\nu\hat{A}_\mu\right) = \left(\nabla_\mu\hat{K}_\nu - \nabla_\nu\hat{K}_\mu\right)\varphi^+ - \varphi\left(\bar{\nabla}_\mu\hat{K}_\nu^+ - \bar{\nabla}_\nu\hat{K}_\mu^+\right).$$

The Yang—Mills lagrangian. The Yang—Mills lagrangian can be written as

$$\begin{aligned} L &= -\frac{1}{2g^2}\,\mathrm{tr}\left(\left(V_{\mu\nu}^\Omega + V_{\mu\nu}^{U_2} + \left[A_\mu, A_\nu\right]\right)^2 + \hat{A}_{\mu\nu}^2\right) \\ &= -\frac{1}{2g^2}\,\mathrm{tr}\left\{C_{\mu\nu}^2 - 4\varphi\varphi^+C_{\mu\nu}^2 + \left(V_{\mu\nu}^\Phi\right)^2 + \left(V_{\mu\nu}^{U_2}\right)^2 + 2V_{\mu\nu}^\Omega V_{\mu\nu}^{U_2} + \left[A_\mu, A_\nu\right]^2\right. \\ &\qquad\qquad \left. + 2\left[A_\mu, A_\nu\right]\left(V_{\mu\nu}^\Omega + V_{\mu\nu}^{U_2}\right) + \hat{A}_{\mu\nu}^2\right\}, \end{aligned} \tag{82}$$

where

$$\begin{aligned} \mathrm{tr}\left(V_{\mu\nu}^\Phi\right)^2 &= \left(-2C_{\mu\nu} + b_{\mu\nu}\right)^2 + \mathrm{tr}\left(2B_{\mu\nu} - R_{\mu\nu}\right)^2 \\ \mathrm{tr}\,B_{\mu\nu}^2 &= \frac{1}{2}\left[\left(\bar{D}_\mu^b\varphi^+D_\nu\varphi\right)\left(\bar{D}_\mu^b\varphi^+D_\nu\varphi\right) - \left(\bar{D}_\mu^b\varphi^+D_\mu\varphi\right)\left(\bar{D}_\nu^b\varphi^+D_\nu\varphi\right)\right] = b_{\mu\nu}^2 \\ \mathrm{tr}\,R_{\mu\nu}^2 &= 2\left(b_\mu\partial_\nu\varphi^+ - b_\nu\partial_\mu\varphi^+\right)\left(b_\mu\partial_\nu\varphi - b_\nu\partial_\mu\varphi\right) \end{aligned}$$

$$
\begin{aligned}
\operatorname{tr} B_{\mu\nu}R_{\mu\nu} &= 4b_\mu^2\partial_\nu\varphi^+\partial_\nu\varphi - 2b_\mu b_\nu\left(\partial_\mu\varphi^+\partial_\nu\varphi + \partial_\nu\varphi^+\partial_\mu\varphi\right)\\
\operatorname{tr}\left[A_\mu, A_\nu\right] V_{\mu\nu}^{\Phi} &= \left(2C_{\mu\nu} - b_{\mu\nu}\right)\operatorname{tr} K_{\mu\nu}\\
&+ \left(K_\mu^+K_\nu - K_\nu^+K_\mu\right)\frac{1}{2}(\bar{D}_\mu^b\varphi^+D_\nu^{bC}\varphi - \bar{D}_\nu^b\varphi^+D_\mu^b\varphi)\\
\operatorname{tr}\left[A_\mu, A_\nu\right] Q_{\mu\nu} &= \operatorname{tr} K_{\mu\nu}Q_{\mu\nu} - \varphi^+Q_{\mu\nu}\varphi\operatorname{tr} K_{\mu\nu}\\
\operatorname{tr}\left[A_\mu, A_\nu\right] S_{\mu\nu} &= 0\\
\operatorname{tr}\left(\hat{A}_{\mu\nu}\right)^2 &= 2\operatorname{tr}\left(\bar{\nabla}_\mu\hat{K}_\nu^+ - \bar{\nabla}_\nu\hat{K}_\mu^+\right)\left(\nabla_\mu\hat{K}_\nu - \nabla_\nu\hat{K}_\mu\right)\\
\operatorname{tr}\ \left[A_\mu, A_\nu\right]^2 &= \operatorname{tr}\left(K_\mu K_\nu^+ - K_\nu K_\mu^+\right)^2 + \left(\operatorname{tr}\left(K_\mu K_\nu^+ - K_\nu K_\mu^+\right)\right)^2
\end{aligned}
$$

Chiral $U(1)$ invariance of CFNS expression. The chiral decomposition G_μ in the CFNS form is invariant under unitary chiral transformations U by definition $G_\mu = V_\mu^U - A_\mu^U$, because G_μ is an U-invariant combination. When we consider only the Yang—Mills sector, this invariance has no consequences. However, when the quark sector is also involved, we should transform chirally quarks and take into account the quantum anomaly of the quark path integral. If the chiral field U does not contain new degrees of fredom compared with the CFNS form, the corresponding quantum anomaly will reflect properties of m, as a common variable of quarks and gluons.

Consider the chiral field

$$U_\chi = \exp im\chi, \partial_\mu\chi = 0, \tag{83}$$

which leaves invariant the CFNS connection $V_\mu^\Omega = mC_\mu + \frac{1}{2}m\partial_\mu m$ in the following sense

$$\left(U_\chi V_\mu^\Omega U_\chi^+ + m\partial_\mu m\right) = V_\mu^\Omega. \tag{84}$$

Then the field $G_\mu = V_\mu^\Omega + Q_\mu - A_\mu$ after the chiral transformation U_χ will become $G_\mu = V_\mu^\chi + Q_\mu - A_\mu^\chi$ with a gauge part V_μ^χ and an axial vector part A_μ^χ

$$V_\mu^\chi = \frac{1}{2}\left(U_\chi\left(V_\mu^\Omega - A_\mu\right)U_\chi^+ + V_\mu^\Omega - A_\mu + U_\chi\partial_\mu U_\chi^+\right) = V_\mu^\Omega - \frac{1}{2}\left(U_\chi A U_\chi^+ + A_\mu\right)$$

$$A_\mu^\chi = \frac{1}{2}\left(A_\mu - U_\chi A_\mu U_\chi^+\right). \tag{85}$$

In the φ-parametrization of A_μ, where $m = 1 - 2\varphi\varphi^+$

$$V_\mu^\chi = V_\mu^\Omega - \cos\chi\left(e^{-i\chi}\varphi\hat{K}_\mu^+ + \hat{K}_\mu\varphi^+e^{i\chi}\right)$$

$$A_\mu^\chi = i\sin\chi\left(e^{-i\chi}\varphi\hat{K}_\mu^+ - \hat{K}_\mu\varphi^+e^{i\chi}\right).$$

It is the interval $\pi/2 \geqslant \chi \geqslant 0$ which is essential for CD: at $\chi = 0$ there is no axial part $\left(V_\mu^\chi, A_\mu^\chi\right) = \left(V_\mu^\Omega - A_\mu, 0\right)$, at $\chi = \pi/2$ the gauge part V_μ^χ is the standard CFN field V_μ^Ω, while $A_\mu^\chi \to A_\mu$.

The hidden $U(1)$ invariance of CFNS decomposition, which does not belong to the gauge group, was found for the $SU(2)$ QCD [13]. This hidden symmetry of CFNS is of chiral origin.

In order to eliminate the superfluous variable, an additional condition is introduced in CFNS [13, 20]

$$D_\mu\left(V^\Omega\right)A_\mu = 0 \tag{86}$$

This condition is covariant under gauge transformations, but breaks U_χ invariance. Indeed, calculating variation of this condition we get for $\delta U_\chi = im\delta\chi$

$$\delta\left\{D_\mu\left(V^\chi\right)A_\mu^\chi\right\} = i\delta\chi\left\{\left(\nabla_\mu K_\mu\varphi^+ - \varphi\bar{\nabla}_\mu\bar{K}_\mu^+\right) + 2K_\mu^+K_\mu\left(\varphi\varphi^+ - 1\right)\right\}, \tag{87}$$

where the first term acts in the CP^2 space, while second one belongs to $U(2)$ space with Q_μ, as a gauge field. At low energies, when dynamics of Q_μ is neglected, the additional condition is equivalent to

$$\nabla_\mu K_\mu = \left(\partial_\mu + \hat{Q}_\mu + b_\mu - 2C_\mu\right)K_\mu = 0. \tag{88}$$

Note, that at the end point $\chi = \pi/2$ the condition (86) is satisfied.

Singlet gluonium and bilinear condensate. We consider gluonium within the framework of the Chiral parametrisation (CP) of the gauge field in QCD $SU(3)$. Acccording to CP, there are two main sectors in QCD: CP^2 sector and $U(2)$ sector. We are interested in gluonium arising in the CP^2 sector containing an abelian field of $SU(3)$. Similar problem has been discussed within the CFNS decomposition in $SU(N)/U(1)^{N-1}$ setting, and the gluonium under consideration was referred as FAG (free abelian gluon). We shall use this name to underline similitude of problems. However, all gluoniums are free and abelian, because they do not interact among themselves. In general, gluoniums are composite particles and they should be described by some kind of collective field. In the case of FAGs, interaction gives only the mass to the free abelian field. In CP the fundamental unit matrix responsible for topology (central in the CFN decomposition) is provided by the quark chiral field. This restricts the form of such matrix and involves quarks in its calculation.

Consider from equation (82) the lagrangian of an Abelian field C_μ. It is convenient to write it in variables $P = \varphi\varphi^+, P_\mu = \partial_\mu P$

$$L(C) = -\frac{1}{2g^2}\left(C_{\mu\nu}^2 - 4C_{\mu\nu}trP\left[P_\mu, P_\nu\right] + trA_{\mu\nu}^2\right) \tag{89}$$

with

$$A_\mu = (1-P)\tilde{K}_\mu P + P\tilde{K}_\mu^+(1-P),$$

where $\tilde{K}_\mu = K_\mu\varphi^+$, $\tilde{K}_\mu^+ = \varphi K_\mu^+$. Note, that there is no cross term $C_{\mu\nu}\left[A_\mu, A_\nu\right]$ in $L(C)$. Second term in L is multiplied by the magnetic field strength $trP\left[P_\mu, P_\nu\right] = b_{\mu\nu}$. Retaining only C_μ as a gauge field in $\hat{A}_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu$ and writing $D_\mu A_\nu = \partial_\mu A_\nu + \left[-2PC_\mu, A_\nu\right]$ we get

$$trA_{\mu\nu}^2 = 2tr_Q\left(\bar{\nabla}_\mu K_\nu^+ - \bar{\nabla}_\nu K_\mu^+\right)\left(\nabla_\mu K_\nu - \nabla_\nu K_\mu\right) = -16C_\mu C_\mu K_\nu^+ K_\nu. \tag{90}$$

It follows from this relation that if there is a space-like vacuum condensate (all fields are anti-hermitian)

$$C_K = \langle K^+K\rangle = \frac{1}{2}\langle \mathrm{tr}\, A_\mu^2\rangle, \tag{91}$$

then the gluonium C_μ can acquire the mass

$$m_C^2 = -2C_K. \tag{92}$$

An existence of dimension $d = 2$ condensate was conjectured from different considerations [32–34]. An obvious candidate for a bilinear condensate is the vacuum value $\langle G_\mu^2\rangle$ of the

QCD gauge field $G_\mu = V_\mu^\Phi - A_\mu$. However, this quantity is not gauge invariant. Therefore, instead of quantity $\langle G_\mu^2 \rangle$ one considers usually $\langle (G_\mu^2)_{\min} \rangle$, a condensate of its minimal value.

In our case, the condensate $\langle K^+ K \rangle$ is a gauge invariant quantity, because A_μ is in an adjoint representation. But we do not know how C_K enters in experimental and lattice data.

Conclusions. We have considered the classical pure gluonic part of QCD and shown that the CFNS decomposition arises quite naturally from the assumption that the basic color unit n-vector of CFNS is a remnant of the quark chiral field and represents a common variable of gluons and quarks. However, the presence of quark chiral DOF's in the CFNS decomposition raises the consistency problem, because the Yang—Mills gauge field does not depend on quarks kinematically. It is this basic assumption that enables usually to consider a decomposition of the QCD gauge field separately, without invoking quarks. Thus, we should show that there are no chiral transformations involving the basic color vector m, which leave the gauge field invariant, but produce change in the quark sector.

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