CONTROL PROCESSES

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T. A. Lepikhin, E. I. Veremey

DIGITAL CONTROL LAWS FOR MARINE AUTOPILOTS WITH INTEGRAL ACTION

St. Petersburg State University, 7/9, Universitetskaya embankment, St. Petersburg, 199034, Russian Federation

In the paper the digital control law with multipurpose structure is considered. The statement of stability for close-loop systems is proved. The integral action of the controllers is considered. The statement in which it is the controller that provides integral action for close-loop system is proved. The principles of MP-structure are considered by two examples of roll feedback control for marine ships and roll stabilization. In the examples the comparison of PID control law and MP-strructure are shown. Bibliogr. 9. Il. 2.

Keywords: digital control law, stability, integral action, multipurpose structure.

Т. А. Лепихин, Е. И. Веремей

ЦИФРОВЫЕ ЗАКОНЫ УПРАВЛЕНИЯ С ОБЕСПЕЧЕНИЕМ АСТАТИЗМА ДЛЯ СТАБИЛИЗАЦИИ КРЕНА МОРСКИХ СУДОВ

Санкт-Петербургский государственный университет, Российская Федерация, 199034, Санкт-Петербург, Университетская наб., 7/9

В статье описывается многоцелевая структура закона управления в цифровом виде. Рассмотрены также подходы к обеспечению астатизма замкнутой системе. Принципы синтеза цифрового закона управления в виде многоцелевой структуры, включающей в себя асимптотический наблюдатель, скоростной регулятор и корректор, продемонстрированы на примерах стабилизации крена судна. Приведено сравнение отработки командных сигналов многоцелевым и ПИД регуляторами. Библиогр. 9 назв. Ил. 2.

Ключевые слова: цифровое управление, астатизм, стабилизация, многоцелевая структура управления.

1. Introduction. Constantly increasing marine traffic determines many problems connected with a safety and reliability of ship motion. To provide these desirable features of sailing, one must use various automatic systems of motion control [1–3], including such commonly used variants as marine autopilots and roll stabilizers.

Lepikhin Timur Andreevich – candidate of physical and mathematical sciences, associate professor; e-mail: LepikhinTA@gmail.com

Veremey Evgeny Igorevich – doctor of physical and mathematical sciences, professor; e-mail: e_veremey@mail.ru

Лепихин Тимур Андреевич – кандидат физико-математических наук, доцент; e-mail: LepikhinTA@gmail.com

Веремей Евгений Игоревич – доктор физико-математических наук, профессор; e-mail: e_veremey@mail.ru

It is well known that a ship motion is extensively influenced by different environmental disturbances such as sea waves, winds, sea currents, change of depth under keel, etc. [1–3]. This determines a vital demand to design control laws for marine applications with the main goal to suppress the mentioned influence of external disturbances as much as possible.

There are a lot of scientific publications devoted to different mathematical approaches to the problem of autopilot design for marine ships of various types [1–4], including course keeping and roll stabilization systems. A complexity of this direction is determined by presence of many dynamical requirements, restrictions, and conditions, to be satisfied by automatic control actions. Now one cannot say that a solution of the problem is exhaustively obtained. As a result, there exist both necessity and capability to develop mathematical methods of control laws synthesis taking into account all desirable features of the closed-loop system's dynamics.

To this end, in contrast to different known approaches, we propose to use ideas based on the theory of multi-purposes control laws synthesis. Fundamentals of this theory were firstly presented in the paper [5] with subsequent transformation in [6] to a modern level. In recent years, some new analytical and numerical methods of synthesis were developed, which are based on the special unified structure of the control laws for marine autopilots.

The mentioned special structure includes some basic part and several additional separate items to be adjusted for an actual environment of sailing. The basic part is invariant with respect to environment, but additional elements can be varied in their dynamics, switched on or off as needed, to provide the best dynamical behavior of the closed-loop system. As it was shown in [5–8], the central item of the structure is the so called dynamical corrector aimed at counteraction of external disturbances.

Here we present a new mathematical and computational method of control synthesis for digital variant of multi-purposes structure for the roll stabilization system. The main attention is paid to the integrity action of control law with respect to external disturbances of the unit step type.

The paper is organized as follows. In Section 2 the special structure of digital control laws is presented and the problem of astatic correction is mathematically posed. Section 3 is devoted to the solution of the posed problem on the base of control law with multi-purposes structure. In Section 4, we discuss peculiarity of providing integrity action for the roll stabilization system. In Section 6 proposed approach is illustrated by the practical example of synthesis for the transport ship with the displacement about 6000 t. Finally, concludes this paper by discussing the overall results of the research.

2. Marine system's control laws with multipurpose structure. Let us consider LTI state space representation for marine ship's mathematical model of the form

$$\begin{aligned} &\mathbf{x}[n+1] = \mathbf{A}\mathbf{x}[n] + \mathbf{B}\delta[n] + \mathbf{H}\mathbf{d}[n], \\ &\delta[n+1] = \mathbf{u}[n] + \delta[n], \\ &\mathbf{y}[n] = \mathbf{C}\mathbf{x}[n] \end{aligned} \tag{1}$$

in discrete time $n \in N^1$. Here $\mathbf{x} \in E^n$ is the state vector, $\delta \in E^m$ is the vector of rudders deflection, vector $\mathbf{d} \in E^l$ presents external disturbances, $\mathbf{y} \in E^k$ is the vector of measured variables, and $\mathbf{u} \in E^m$ is the vector of controls.

All matrices $\mathbf{A}, \mathbf{B}, \mathbf{H}$ and $\mathbf{C}$ with constant components have correspondent dimensions. Suppose that system (1) is controllable by $\mathbf{u}$ and observable by $\mathbf{y}$.

In general, stabilizing control law for the plant (1) can be presented as

$$\mathbf{u} = \mathbf{W}(q)\mathbf{y} + \mathbf{W_0}(q)\delta, \tag{2}$$

where q denotes backward shift operator in discrete time. Transfer matrices $\mathbf{W}$ and $\mathbf{W_0}$ of the controller (2) are not given initially: as a rule, they should be searched as a solution of the optimization problem

$$J = J(\mathbf{W}, \mathbf{W_0}) \to \inf_{\mathbf{W}, \mathbf{W_0} \in \Omega}. \tag{3}$$

Here functional J reflects our treatment of the quality of external disturbances suppression for the closed-loop system (1), (2). An admissible set $\Omega^* \subset \Omega$ of the controllers (2) is a restriction of the stabilizing controllers set Ω. We shall determine this restriction here by the requirement of integral action of controllers (2) with initially given multi-purposes (MP) structure. Some algorithms for the problem (3) solving are discussed in [9].

Definition 1. Given a stable closed-loop system (1), (2), we say that this one is astatic with respect to the input $\mathbf{d}$ and output $\mathbf{y}$ if for any $\mathbf{d_0} \in E^l$ we have $\lim_{n\to\infty}\{\|\mathbf{y_d}[n]\|\} = 0$, where $\mathbf{y_d} = \{\mathbf{y_d}[n]\}$ is system response to the input step $\mathbf{d} = \{\mathbf{d}[n]\}, \mathbf{d}[n] = \mathbf{d_0} \cdot 1[n]$. We also say that the correspondent controller (2) provides an integral action for this closed-loop connection.

The main problem to be considered in this paper is to design a controller (2) with MP-structure, providing its integral action, and optimizing its parameters in accordance with (3). As for the performance index J, we accept that the quality of the control is determined by maximum deflection of the output norm $\|\mathbf{y_d}[n]\|$ for the mentioned process with the given vector $\mathbf{d_0}$.

Definition 2. Given a controlled plant (1), we say that the controller (2) has MP-structure if its mathematical model consists of the following three items:

a) asymptotic observer

$$\mathbf{z}[n+1] = \mathbf{A}\mathbf{z}[n] + \mathbf{B}\delta[n] + \mathbf{G}(\mathbf{y}[n] - \mathbf{C}\mathbf{z}[n]); \tag{4}$$

b) dynamical corrector

$$\xi = \mathbf{F}(q)(\mathbf{y} - \mathbf{C}\mathbf{z}), \mathbf{F}(q) = [\alpha, \beta, \gamma, \mathbf{0}]; \tag{5}$$

c) observer-based corrected output

$$\mathbf{u}[n] = \mathbf{K_x}\mathbf{z}[n] + \mathbf{K}_\delta\delta[n] + \xi[n]. \tag{6}$$

Remark that controller (4)–(6) with MP-structure has inputs $\mathbf{y}[n]$, $\delta[n]$, output $\mathbf{u}[n]$, and state $\mathbf{z}[n]$ for every instant of discrete time n. The main feature of this controller is determined by the following statement.

Theorem 1. *If matrices* $\mathbf{A} - \mathbf{GC}$, α, *and* $\left(\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{K}_x & \mathbf{E_m} + \mathbf{K}_\delta \end{array}\right)$ *are Schur ones, i.e. if observer, corrector and state-driving plant* (*by the controller* $\mathbf{u} = \mathbf{K_x}\mathbf{x} + \mathbf{K}_\delta\delta$) *are asymptotically stable, then the closed-loop connection* (1), (4)–(6) *is also asymptotically stable.*

Proof. Let us consider mathematical model of the mentioned closed-loop system with zero initial conditions:

$$\begin{array}{l} \mathbf{x}[n+1] = \mathbf{A}\mathbf{x}[n] + \mathbf{B}\delta[n] + \mathbf{H}\mathbf{d}[n], \\ \delta[n+1] = (\mathbf{E_m} + \mathbf{K}_\delta)\delta[n] + \mathbf{K_x}\mathbf{z}[n] + \gamma\mathbf{r}[n], \\ \mathbf{z}[n+1] = \mathbf{GC}\mathbf{x}[n] + \mathbf{B}\delta[n] + (\mathbf{A} - \mathbf{GC})\mathbf{z}[n], \\ \mathbf{r}[n+1] = \beta\mathbf{C}\mathbf{x}[n] - \beta\mathbf{C}\mathbf{z}[n] + \alpha\mathbf{r}[n], \end{array} \tag{7}$$

where $\mathbf{r} \in \mathbf{E^{n_1}}$ is the state vector of the corrector. We can write the characteristic polynomial of this system as

$$\Delta(z) = \begin{vmatrix} \mathbf{E}z - \mathbf{A} & -\mathbf{B} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E_m}z - \mathbf{E}_m - \mathbf{K}_\delta & -\mathbf{K}_x & -\gamma \\ -\mathbf{GC} & -\mathbf{B} & \mathbf{E}z - \mathbf{A} + \mathbf{GC} & \mathbf{0} \\ -\beta\mathbf{C} & \mathbf{0} & \beta\mathbf{C} & \mathbf{E_{n_1}}z - \alpha \end{vmatrix}.$$

It is a matter of simple calculation to verify that the determinant here can be transformed as follows:

$$\Delta(z) = \begin{vmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{K}_x & \mathbf{E}_m + \mathbf{K}_\delta \end{vmatrix} \cdot det(\mathbf{K}z - \mathbf{A} + \mathbf{GC}) \cdot det(\mathbf{E_{n_1}}z - \alpha)$$

that immediately proves the theorem 1.

Note that polynomial $\Delta(z)$ does not depend on matrices β and γ of the corrector. This gives us an easy possibility to choose these matrices, providing desirable dynamical features of the close-loop system. In particular, we can design a corrector to achieve integral action of controller (4)–(6); this is the matter of the next consideration.

3. Integral action of the controllers. The main goal of the discussion here is to obtain conditions of integral action for the controllers with MP-structure.

To begin with, let us initially refer to the plant (1), closed by the controller (2) in general form. It is well known that any asymptotically stable DLTI system with tf-model $\mathbf{y} = \mathbf{T}(z)\mathbf{d}$ is astatic if and only if the equality $\mathbf{T}(1) = \mathbf{0}$ holds. On the base of this condition, we arrive at the following statement.

Lemma 1. *For the closed-loop connection* (1), (2) *to be astatic it is necessary and sufficient to provide the equality*

$$\mathbf{C}[\mathbf{E} - \mathbf{A} + \mathbf{B}\mathbf{W}_\delta^{-1}(1)\mathbf{W_x}(1)]^{-1}\mathbf{H} = \mathbf{0}. \tag{8}$$

Proof. Let us use z-transformation to write the equations of the system under zero initial conditions: $z\mathbf{x} = \mathbf{Ax} + \mathbf{B}\delta + \mathbf{H}\varphi$, $z\delta = \mathbf{u} + \delta$, $\mathbf{u} = \mathbf{W_x}(z)\mathbf{x} + \mathbf{W}_\delta(z)\delta$, $\mathbf{y} = \mathbf{Cx}$. This implies that $\mathbf{y} = \mathbf{C}\{\mathbf{E}z - \mathbf{A} - \mathbf{B}[\mathbf{E_m}z - \mathbf{E_m} - \mathbf{W}_\delta(z)]^{-1}\mathbf{W_x}(z)\}^{-1}\mathbf{Hd}$, where $\mathbf{E_m}$ is $m \times m$ identity matrix. Consequently, the transfer matrix of the closed-loop system is

$$\mathbf{T}(z) = \mathbf{C}\{\mathbf{E}z - \mathbf{A} - \mathbf{B}[\mathbf{E_m}z - \mathbf{E_m} - \mathbf{W}_\delta(z)]^{-1}\mathbf{W_x}(z)\}^{-1}\mathbf{H},$$

and from $\mathbf{T}(z) = \mathbf{0}$ we have (8).

Let us consider one practically significant particular situation for the state-driving plant (1) with the controller

$$\mathbf{u}[n] = \mathbf{K_x}\mathbf{x}[n] + \mathbf{K}_\delta\delta[n], \tag{9}$$

here matrices $\mathbf{K_x}, \mathbf{K}_\delta$ are such that matrix $\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{K}_x & \mathbf{E}_m + \mathbf{K}_\delta \end{pmatrix}$ is Schur.

Let us introduce an additional assumption that

$$\dim \mathbf{y} = \dim \delta = m; \quad rank\ \mathbf{C} = m. \tag{10}$$

From the last equality follows that we can present the output equation as

$$\mathbf{y} = \mathbf{C}_a\mathbf{x}_a + \mathbf{C}_p\mathbf{x}_p, \quad \mathbf{C} = (\mathbf{C}_a\ \mathbf{C}_p), \quad \mathbf{x} = (\mathbf{x_a'}\ \mathbf{x_p'})', \tag{11}$$

where $\dim \mathbf{x}_p = m$, $\dim \mathbf{x}_a = n_z - m$, and $m \times m$ matrix $\mathbf{C}_p$ is not singular; from (11) we obtain $\mathbf{x}_p = \mathbf{C}_p^{-1}\mathbf{p} - \mathbf{C}_p^{-1}\mathbf{C}_a\mathbf{x}_a$. Then let us rewrite plant equation (1) supposing $\mathbf{d} \equiv \mathbf{0}$ as follows:

$$\mathbf{x}[n+1] - \mathbf{x}[n] = (\mathbf{A} - \mathbf{E})\mathbf{x}[n] + \mathbf{B}\delta[n]. \tag{12}$$

In accordance with the dimensions of vectors $\mathbf{x}_a$ and $\mathbf{x}_p$ let partition the matrices $\mathbf{A} - \mathbf{E}$ and $\mathbf{K}_\mathbf{x}$ into correspondent blocks: $\mathbf{A} - \mathbf{E} = (\mathbf{A}_a \ \mathbf{A}_p)$, $\mathbf{K}_\mathbf{x} = (\mathbf{K}_{xa} \ \mathbf{K}_{xp})$.

We can present the equation (12) on the base of introduced notations as follows:

$$\mathbf{x}[n+1] - \mathbf{x}[n] = (\mathbf{A}_a - \mathbf{A}_p\mathbf{C}_p^{-1}\mathbf{C}_a)\mathbf{x}_a[n] + \mathbf{B}\delta[n] + \mathbf{A}_p\mathbf{C}_p^{-1}\mathbf{y}[n].$$

Introducing an auxiliary vector $\gamma = (\mathbf{x}_\mathbf{a}' \ \delta')' \in \mathbf{E}^n$ and $n_s \times n_s$ matrix

$$\mathbf{S} = \begin{pmatrix} \mathbf{A}_a & -\mathbf{A}_p\mathbf{C}_p^{-1}\mathbf{C}_a & \mathbf{B} \end{pmatrix},$$

obtain the plant equation of the form

$$\mathbf{x}[n+1] - \mathbf{x}[n] = \mathbf{S}\gamma[n] + \mathbf{A}_p\mathbf{C}_p^{-1}\mathbf{y}[n]. \tag{13}$$

We can provide an integral action of the controller with the help of the following statement.

Theorem 2. *If we have no external disturbance for the plant* (1) *and if matrix* $\mathbf{S} = (\mathbf{A}_a \ \ - \mathbf{A}_p\mathbf{C}_p^{-1}\mathbf{C}_a \ \ \mathbf{B})$ *is not singular then the controller* (9) *can be presented in the following equivalent form*:

$$\mathbf{u}[n] = \mu(\mathbf{x}[n+1] - \mathbf{x}[n]) + \nu\mathbf{y}[n], \tag{14}$$

where $\mu = \mathbf{MS}^{-1}$; $\nu = \mathbf{K}_{xp}\mathbf{C}_p^{-1} - \mathbf{MS}^{-1}\mathbf{A}_p\mathbf{C}_p^{-1}$; $\mathbf{M} = (\mathbf{K}_{xa} \ \ - \mathbf{K}_{xp}\mathbf{C}_p^{-1}\mathbf{C}_a \ \ \mathbf{K}_\delta)$.

Moreover, if the matrix ν is also not singular, then the controller (14) provides an astatic feature for the closed-loop system

$$\begin{aligned} &\mathbf{x}[n+1] = \mathbf{A}\mathbf{x}[n] + \mathbf{B}\delta[n] + \mathbf{H}\mathbf{d}[n], \\ &\delta[n+1] = (\mathbf{u})[n] + \delta[n], \\ &\mathbf{u}[n] = \mu(\mathbf{x}[n+1] - \mathbf{x}[n]) + \nu\mathbf{y}[n], \\ &\mathbf{y} = \mathbf{C}\mathbf{x}. \end{aligned} \tag{15}$$

Proof. Supposing $\det \mathbf{S} \neq 0$, we can obtain vector γ from (13):

$$\gamma = \mathbf{S}^{-1}(\mathbf{x}[n+1] - \mathbf{x}[n]) - \mathbf{S}^{-1}\mathbf{A}_p\mathbf{C}_p^{-1}\mathbf{y}. \tag{16}$$

Now let us transform the controller (9) to the form

$$\mathbf{u} = \mathbf{K}_x\mathbf{x} + \mathbf{K}_\delta\delta = \mathbf{K}_{xa}x_p + \mathbf{K}_{xp}x_p + \mathbf{K}_\delta\delta = \mathbf{M}\gamma + \mathbf{K}_{xp}\mathbf{C}_p^{-1}\mathbf{y},$$

and after substitution (16) we obtain equivalent control law

$$\mathbf{u}[n] = \mu(\mathbf{x}[n+1] - \mathbf{x}[n]) + \nu\mathbf{y}[n], \tag{17}$$

based on the back differences of the state vector components.

Using Lemma 1, one can verify that controller (17) provides an integral action for system (15). Nevertheless, this verification can be realized in a more simple way. Really, let us consider the equation of actuator jointly with the controller (17):

$$\delta[n+1] - \delta[n] = \mu(\mathbf{x}[n+1] - \mathbf{x}[n]) + \nu\mathbf{y}. \tag{18}$$

If the closed-loop system (15) has an equilibrium position under action of any step external disturbance $\mathbf{d}$, then for this position the following equalities hold:

$$\mathbf{x}[n+1]-\mathbf{x}[n]=\mathbf{0}, \quad \delta[n+1]-\delta[n]=\mathbf{0}, \quad \forall n \in N^1,$$

and in correspondence with (18) we have $\nu\mathbf{y}=0$. Because of the matrix ν is not singular, this linear system has the unique solution $\mathbf{y}=0$, i.e. the closed-loop system (15) is astatic.

At last, let us apply a controller with MP-structure of the form

$$\begin{aligned}
&\mathbf{z}[n+1]=\mathbf{A}\mathbf{z}[n]+\mathbf{B}\delta[n]+\mathbf{G}(\mathbf{y}[n]-\mathbf{C}\mathbf{z}[n]),\\
&\xi=\mathbf{F}(q)(\mathbf{y}-\mathbf{C}\mathbf{z}),\\
&\mathbf{F}(q)=[\alpha,\beta,\gamma,\mathbf{0}],\\
&\mathbf{u}[n]=\mu(\mathbf{z}[n+1]-\mathbf{z}[n])+\nu\mathbf{y}[n]+\xi[n],
\end{aligned} \tag{19}$$

where, in contrast with (4)–(6), corrected output is changed by analogy to (17). The following statement determines integral action of the controller (19).

Theorem 3. *If a choice of the matrices $\mathbf{K_x}$, $\mathbf{K}_\delta$, $\mathbf{G}$, and α provides an asymptotic stability of the closed-loop system* (7)*, if the matrices μ and ν are determined by formula* (14)*, and if additionally equality $\mathbf{F}(1)=\mathbf{0}$ holds, then this system is astatic with respect to input $\mathbf{d}$ and output $\mathbf{y}$.*

Proof. First, let us prove that the closed-loop connection (1), (19) is also asymptotically stable. To this end, let refer to the formula (14) and obviously obtain

$$\mu(\mathbf{A}-\mathbf{E})+\nu\mathbf{C}=\mathbf{K_x}, \quad \mu\mathbf{B}=\mathbf{K}_\delta. \tag{20}$$

Taking into account the following identity:

$$\mathbf{z}[n+1]-\mathbf{z}[n]=(\mathbf{A}-\mathbf{E})\mathbf{z}[n]+\mathbf{B}\delta[n]+\mathbf{GC}(\mathbf{x}[n]-\mathbf{C}\mathbf{z}[n]),$$

we have

$$\begin{aligned}
&\mathbf{u}[n]=\mu(\mathbf{z}[n]-\mathbf{z}[n])+\nu\mathbf{y}[n]+\gamma\mathbf{r}[n]=\\
&=\mu\{(\mathbf{A}-\mathbf{E})\mathbf{z}[n]+\mathbf{B}\delta[n]+\mathbf{GC}(\mathbf{x}[n]-\mathbf{C}\mathbf{z}[n])\}+\nu\mathbf{C}\mathbf{x}[n]+\gamma\mathbf{r}[n].
\end{aligned} \tag{21}$$

Using the control signal (21) we can present mathematical model of the closed-loop connection (1), (19) as follows:

$$\begin{aligned}
\mathbf{x}[n+1]&=\mathbf{A}\mathbf{x}[n]+\mathbf{B}\delta[n]+\mathbf{H}\mathbf{d}[n],\\
\delta[n+1]&=(\mu\mathbf{GC}+\nu\mathbf{C})\mathbf{x}[n]+(\mu\mathbf{B}+\mathbf{E}_m)\delta[n]+\mu(\mathbf{A}-\mathbf{E}-\mathbf{GC})\mathbf{z}[n]+\gamma\mathbf{r}[n],\\
\mathbf{z}[n+1]&=\mathbf{GC}\mathbf{x}[n]+\mathbf{B}\delta[n]+(\mathbf{A}-\mathbf{GC})\mathbf{z}[n],\\
\mathbf{r}[n+1]&=\beta\mathbf{C}\mathbf{x}[n]-\beta\mathbf{C}\mathbf{z}[n]+\alpha\mathbf{r}[n].
\end{aligned} \tag{22}$$

By the analogy with the proof of Theorem 1, we can write the characteristic polynomial of this system as

$$\Delta(z)=\begin{vmatrix}
\mathbf{E}z-\mathbf{A} & -\mathbf{B} & \mathbf{0} & \mathbf{0}\\
-\mu\mathbf{GC}-\nu\mathbf{C} & \mathbf{E_m}z-\mathbf{E}_m-\mu\mathbf{B} & -\mu(\mathbf{A}-E-GC) & -\gamma\\
-\mathbf{GC} & -\mathbf{B} & \mathbf{E}z-\mathbf{A}+\mathbf{GC} & \mathbf{0}\\
-\beta\mathbf{C} & \mathbf{0} & \beta\mathbf{C} & \mathbf{E_{n_1}}z-\alpha
\end{vmatrix}.$$

Then, taking into account the equalities (20), one can easyly obtain representation

$$\Delta(z)=\begin{vmatrix}
\mathbf{A} & \mathbf{B}\\
\mathbf{K}_x & \mathbf{E}_m+\mathbf{K}_\delta
\end{vmatrix}\cdot\det(\mathbf{E}z-\mathbf{A}+\mathbf{GC})\cdot\det(\mathbf{E}_{n_1}z-\alpha)$$

that confirms asymptotic stability.

Next, let us use equation of the actuators jointly with control signal $\mathbf{u}$:

$$\delta[n+1] - \delta[n] = (\mu(\mathbf{z}[n+1] - \mathbf{z}[n]) + \nu \mathbf{y}[n] + \mathbf{F}(q)\mathbf{C}(\mathbf{x}[n] - \mathbf{z}[n])). \tag{23}$$

As well as for Theorem 2, if the closed-loop system (22) has an equilibrium position under action of the any step external disturbance $\mathbf{d}$, then for this position the following equalities hold:

$$\mathbf{x}[n+1] - \mathbf{x}[n] = \mathbf{0}, \quad \delta[n+1] - \delta[n] = \mathbf{0}, \quad \forall n \in N^1.$$

So, in accordance with (23), taking into account $\mathbf{F}(1) = \mathbf{0}$, we obviously obtain $\nu \mathbf{y} = \mathbf{0}$. Because we have supposed that the matrix ν is not singular, this linear system has the unique solution $\mathbf{y} = 0$, i.e. the closed-loop system (22) is astatic. End proof.

One can observe that the proven theorems determine some freedom of the choice for the items of MP-structure. This allows realizing integral action of the controller and additionally optimizing (in the sense of (3)) a closed-loop connection sequentially by the coefficients of the base controller, the matrix of observer, and the transfer matrix of the corrector.

4. Roll feedback control for marine ships. Modern marine roll stabilization systems have a significant impact on performance of ships and various marine structures allowing them to perform their mission in severe sea conditions during long periods of time [3]. These systems should be designed and constructed to avoid large values of the roll angle in any environmental conditions of sailing. In particular, this motivates a desire to synthesize control laws with integral action to stand up against step type external disturbances. Let us accept the simplest equations [3], presenting roll motion of the ship as follows:

$$\dot{\omega} = \frac{N_x}{J_{xx}(1 + k_{44})}, \quad \dot{\theta} = \omega. \tag{24}$$

In formula (24) θ is the roll angle, ω is the roll rate. Variable N_x presents the generalized roll torque, which can be calculated by formula

$$N_x = -m_s J_{xx} \omega - mgh_0 \theta + c_1 V^2 \delta + M_b + M_x,$$

where $M_b = c_2 V^2 \delta - c_3 V \omega$ is the moment determined by actions of the board roll wings; δ is the angle of wing's deflection; M_x is the torque from external forces; V is the constant speed of the motion; J_{xx} is the moment of ship's inertia. Parameters m, m_s, h_0, c_1, c_2 and c_3 are given constants depending on constructive features of the specific ship.

LTI mathematical model of a ship's roll motion in continuous time is as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + d(t), \tag{25}$$

where $\mathbf{x} = (\omega \;\; \theta \;\; \delta)'$ is the state vector, $\mathbf{A}$ and $\mathbf{b}$ are the matrices of the form

$$\mathbf{A} = \begin{pmatrix} a_1 & a_2 & b \\ 1 & 0 & 0 \\ 0 & 0 & -k_g \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ k_g \end{pmatrix}$$

with constant components a_1, a_2, k_g.

Let us obtain correspondent DLTI model in discrete time after discretization by the Euler method with the constant sample period Δt:

$$\mathbf{x}[n+1] = \mathbf{A}_d \mathbf{x}[n] + \mathbf{b}_d u[n] + d_d[n], \tag{26}$$

here $\mathbf{A}_d = \mathbf{E}_{3\times 3} + \Delta t \mathbf{A}, \quad \mathbf{b}_d = \Delta t \mathbf{b} u$, $d_d[n] = \Delta t d(n\Delta t)$.

As for the measured and controlled variable y, we accept the roll angle as the system output:

$$y = \mathbf{c}\mathbf{x}, \quad \mathbf{c} = (0 \;\; 1 \;\; 0). \tag{27}$$

Let us apply controller (19) with MP-structure for the roll stabilization system on the base of approach proposed above. Note that for this particular case one can present the mentioned controller as follows:

$$\begin{aligned} &z_1[n+1] = z_1[n] + \Delta t a_1 z_1[n] + \Delta t a_2 z_2[n] + \Delta t b\delta[n] + g_1(\theta[n] - z_2[n]), \\ &z_2[n+1] = z_2[n] + \Delta t z_1[n] + g_2(\theta[n] - z_2[n]), \\ &\xi = F(q)(\theta[n] - z_2[n]), \\ &u = \mu_1(z_1[n+1] - z_1[n]) + \mu_2(z_2[n+1] - z_2[n]) + \nu\theta + \delta + \xi. \end{aligned} \tag{28}$$

To calculate unknown parameters of the controller (28), we can apply the following sequential scheme in accordance with the properties and features of MP-structure discussed in the preceding sections.

1. First, it is necessary to find the base state controller of the form

$$u = k_1\omega + k_2\theta + k_3\delta \equiv \mathbf{k}_c\mathbf{x}, \quad \text{where } \mathbf{k}_c = (k_1 \;\; k_2 \;\; k_3). \tag{29}$$

This one is used as a state-driving initial feedback for the controlled plant (26) providing stability and desirable dynamical features of the closed-loop connection (26), (29). For example, the state controller can be obtained as a solution of well-known LQR-optimization problem, or as a solution of the problem (3).

2. Secondly, one needs to recalculate obtained parameters k_1, k_2, k_3 into the coefficients μ_1, μ_2, and ν, using formulas (10), (11), and (14).

3. Thirdly, we have to obtain coefficients g_1 and g_2 of the asymptotic observer. This item of the scheme is aimed to providing observer stability and to guarantee desirable dynamical features of the closed-loop system under actions of the step-type external disturbances. In particular case, Kalman filter is quite suitable to be used for these purposes; another variant is also based on the solution of the problem (3).

4. The last position of the scheme consists of the corrector adjustment, i.e. we need to find a transfer function $F(q)$ of the corrector. This one must have Schur denominator and should provide desirable dynamics of the closed-loop system under actions of sea wave external disturbances. The simplest variant of the correction is its full absence in the controller structure. The most complicated one assumes that the transfer function $F(q)$ is a solution of the correspondent problem (3): this is a subject of detailed discussion presented in the paper [7]. Hear we restrain our consideration only by the integral action of the controller.

5. Roll stabilization example. Let us illustrate a practical implementation of the scheme presented above by the example of the roll stabilizer synthesis for the transport ship with the displacement about 6000 t.

Assume that we have given mathematical model (25), (27) of the roll ship motion with the following values of the parameters:

$$a_1 = -0.150, \quad a_2 = -0.360, \quad b = -0.0480, \quad k_g = 15.$$

It is necessary to take into account that deflections of the board wings and the rate of these deflections are limited by the following values correspondently:

$$\delta_{\max} = 30^\circ, \quad \dot{\delta}_{\max} = 35^\circ/\text{s}.$$

Let us accept the sampling period $\Delta t = 0.5$ s for the discrete model (26).

In accordance with the first step of the scheme, we can pose the problem (3) for the plant (26) closed by the controller

$$u = k_1\omega + k_2(\theta - \theta^*) + k_3\delta \equiv \mathbf{k}_c\mathbf{x} - k_2\theta^*, \tag{30}$$

where $\theta^* = 3^\circ$ is given reference signal for the roll angle. The functional J in (3) is presented by the roll overshoot, and admissible set of the vectors $\mathbf{k}_c$ is determined by desirable stability degree $ds(\mathbf{k}_c) \leqslant 0.95$ of the closed-loop connection.

As a result of optimization, we obtain the following coefficients of the base controller (29): $k_1 = 11.3$, $k_2 = -0.200$, $k_3 = -0.0355$. Correspondent transient process for this controller is presented on the fig. 1, where command signal is determined as $\delta = -[k_1\omega + k_2(\theta - \theta^*)]k_3$.

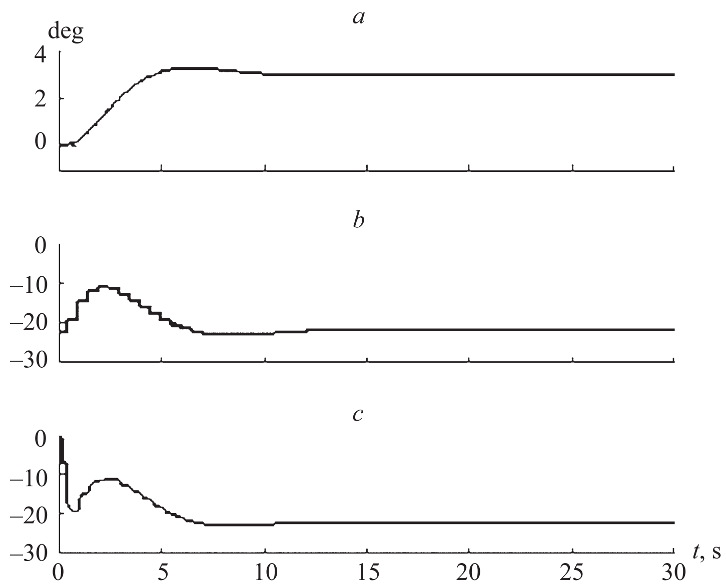

Fig. 1. Transient process for the system (26), (30)
a – roll angle; *b* – command signal; *c* – rudder deflection.

Simple computations in accordance with the formulas (10), (11), and (14) allow to obtain the following coefficients μ and ν for the controller (28): $\mu_1 = 21.6$, $\mu_2 = 14.5$, $\nu = 7.57$.

Then, by analogy, we obtain coefficients $g_1 = 0.363$, $g_2 = 1.85$ of the asymptotic observer.

Finally, supposing that the sailing is not disturbed by sea waves, we can switch off a corrector, i.e. accept $F(q) \equiv 0$.

To demonstrate the results of synthesis, let us consider stabilization process for the closed-loop system under influence of the step-type external disturbance $d(t) = d_0 \cdot 1(t)$, $d_0 = 1$. For the comparison, we use PID-controller of the form

$$u = k_1\omega + k_2\theta + k_3\delta + k_i \int\limits_0^t \theta(\tau)d\tau = \mathbf{k}_c\mathbf{x} + k_i\mathbf{C} \int\limits_0^t \mathbf{x}(\tau)d\tau \tag{31}$$

for the same process $(k_i) = 1.5$.

Stabilization processes for the closed-loop systems (26), (28) and (26), (31) correspondently are illustrated by fig. 2.

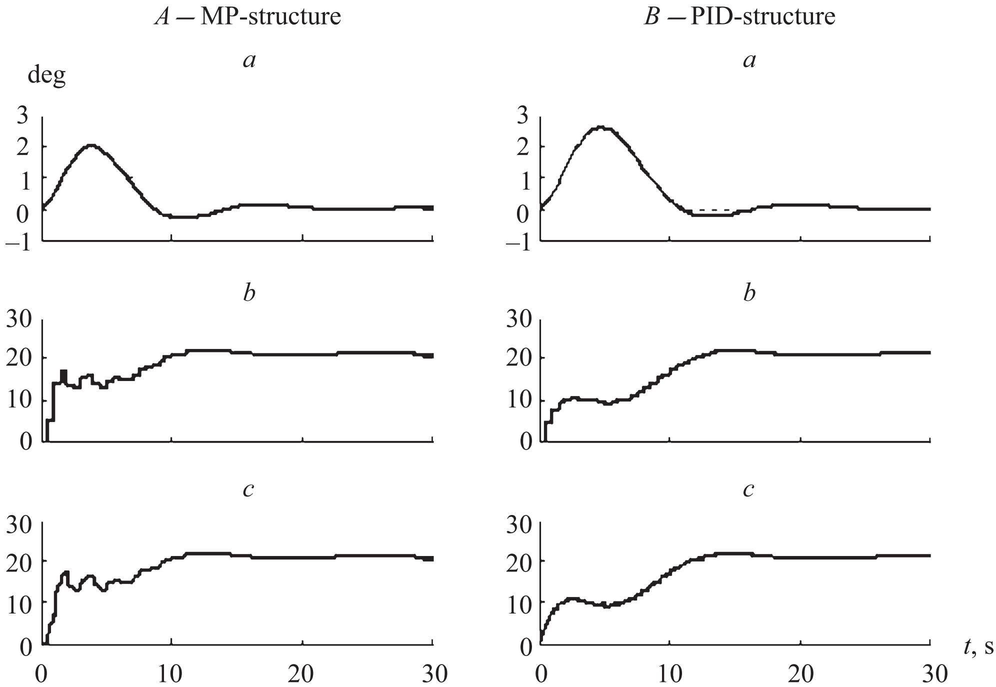

Fig. 2. Stabilization processes for the step-type external disturbance
a – roll angle; *b* – command signal; *c* – rudder deflection.

The comparison of the presented processes shows that the synthesized controller (28) with MP-structure provides 25% decrease of the settling time and overshoot with respect to the controller (31) with traditional PID-structure.

6. Conclusions. The main goal of this work is to propose a constructive method of marine roll stabilizer synthesis to provide an astatic property of the closed-loop connection. In contrast to well-known approaches, usually based on PID-structures of control laws, we achieve this goal using a controller with the special multi-purposes structure. In our opinion, this method provides a certain flexibility of the control law with respect to an actual environment of sailing. The mentioned dynamical features of the control law (19) with MP-structure grant such flexibility. Here we can select the considered corrective term subject to a current regime of the ship motion in the following variants:

1. If a ship moves under condition of quiet water, we can fully switch off the dynamical corrector in control law and use simplified controller (4), (6) with no astatic feature working in the spared regime.

2. If we have a motion under significant step-type disturbances, but no sea wave, it is quite suitable to accept the control law (19) with no corrector, providing an integral action of the controller that does not overload the system by additional useless dynamics.

3. If sea wave also influences to the ship motion, we can switch on the corrector for the controller (19), using them in accurate roll stabilization regime [8], and keeping an integral property to react against step-type disturbances.

4. At last, to use economical regime of stabilization, we can change a transfer matrix of the corrector for the notch filtering action in accordance with recommendations [7], also keeping integral action of the controller.

We believe that the approach proposed here can be useful not only for the surface ships, but also for various kinds of AUVs and flying offshore structures roll stabilizers design. The results of investigations presented above can be developed to take into account transport delays and robust features of the control law.

References

1. Fossen T. I. Guidance and control of ocean Vehicles. New York: John Wiley and Sons, 1989, 480 p.
2. Fossen T. I. Handbook of marine craft hydrodynamics and motion control. New York: John Wiley and Sons, Ltd., 2011, 600 p.
3. Perez T. Ship Motion Control: Course Keeping and Roll Stabilization using rudder and fins. London: Springer-Verlag, 2005, 300 p.
4. Dove M. J., Wright C. B. Development of Marine Autopilots. *Computer Methods in Marine and Offshore Engineering*. Southampton: Computational Mechanics Publ., 1991, pp. 259–272.
5. Veremei E. I., Korchanov V. M. Multiobjective stabilization of a certain class of dynamic systems. *Automation and Remote Control*, 1989, vol. 49, no. 9, pp. 1210–1219.
6. Veremey E. I. Synthesis of multiobjective control laws for ship motion. *Gyroscopy and Navigation*, 2010, vol. 1, no. 2, pp. 119–125.
7. Veremey E. I. H_∞-Approach to wave disturbance filtering for marine autopilots. *Proc. of the 9th IFAC Conference on manoeuvring and control of marine craft* (MCMC 2012). Arenzano, Italy, September 19–21, 2012–2013, pp. 410–415.
8. Veremey E. I. Dynamical Correction of Control Laws for Marine Ships' Accurate Steering. *Journal of Marine Science and Application*, 2014, vol. 13, no. 2, pp. 127–133 (doi: 10.1007/s11804-014-1250-1).
9. Veremey E. I. Algorithms for Solving a Class of Problems of H_∞-optimization of Control Systems. *Journal of Computer and Systems Sciences International*, 2011, vol. 50, no. 3, pp. 403–412.

Литература

1. *Fossen T. I.* Guidance and control of ocean Vehicles. New York: John Wiley and Sons, 1989. 480 p.
2. *Fossen T. I.* Handbook of marine craft hydrodynamics and motion control. New York: John Wiley and Sons, Ltd., 2011. 600 p.
3. *Perez T.* Ship Motion Control: Course Keeping and Roll Stabilization using rudder and fins. London: Springer-Verlag, 2005. 300 p.
4. *Dove M. J., Wright C. B.* Development of Marine Autopilots // Computer Methods in Marine and Offshore Engineering. Southampton: Computational Mechanics Publ., 1991. P. 259–272.
5. *Veremei E. I., Korchanov V. M.* Multiobjective stabilization of a certain class of dynamic systems // Automation and Remote Control. 1989. Vol. 49, N 9. P. 1210–1219.
6. *Veremey E. I.* Synthesis of multiobjective control laws for ship motion // Gyroscopy and Navigation. 2010. Vol. 1, N 2. P. 119–125.
7. *Veremey E. I.* H_∞-Approach to wave disturbance filtering for marine autopilots // Proc. of the 9th IFAC Conference on manoeuvring and control of marine craft (MCMC 2012). Arenzano, Italy, September 19–21, 2012–2013. P. 410–415.
8. *Veremey E. I.* Dynamical Correction of Control Laws for Marine Ships' Accurate Steering // Journal of Marine Science and Application. 2014. Vol. 13, N 2. P. 127–133 (doi: 10.1007/s11804-014-1250-1).
9. *Veremey E. I.* Algorithms for Solving a Class of Problems of H_∞-optimization of Control Systems // Journal of Computer and Systems Sciences International. 2011. Vol. 50, N 3. P. 403–412.

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